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Robust Direction of Arrival Estimation Using Uniform Circular Antenna Array based on Total Forward - Backward Matrix Pencil Method

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***Abstract*—In this paper, we propose an approach to estimate the Direction of Arrival (DOA) of Radio coherent incoming signals using the Total Forward – Backward Matrix Pencil algorithm (TFBMP). This algorithm works directly on samples of signals impinging on an M – element Uniform Circular Antenna (UCA) array, which has a smaller size as well as larger observation angle in comparison with the Uniform Linear Antenna (ULA) array. Therefore, the correlation between the received signals does not significantly impact on its performance and efficiency. Furthermore, this algorithm can also extract the DOA information with only one snapshot of signal. Simulation results for DOA estimation using the proposed approach for different situations of incoming signals as well as the number of snapshots in the presence of noise will be assessed to verify its performance.**

Keywords* - *Direction of Arrival (DOA), Total Forward - Backward Matrix Pencil (TFBMP), Uniform Circular Antenna (UCA) Array.

I. INTRODUCTION

Radio Direction Finding (DF) is a technique that identifies the bearing angle or the coordinates of incoming radio signals. The most important information estimated by a DF system is the Direction of Arrival (DOA). DF systems have many applications in Radio Navigation, Emergency Aid and intelligent operations, etc. Several techniques to calculate the DOA information have been investigated. They can be classified into two main categories: amplitude comparison based [1] and phase comparison based [2], [3]. Some other algorithms can be the hybrid of the two such as the Correlative vector (CV) method [4].

In [5 - 8], the Matrix Pencil algorithm was proposed as a high – resolution technique for DOA estimation. Matrix Pencil has some advantages in comparison with the other super – resolution methods for DOA estimation such as MUSIC [9], which generally has to calculate the signal covariance matrix. Unlike other algorithms, Matrix Pencil works directly on signal samples and does not require independent data samples. Furthermore, the Matrix Pencil algorithm offers some benefits such as less processing power and faster executing than some other super – resolution methods [7]. One of the most remarkable advantages of this technique is that it can extract the DOA information with one snapshot.

Total Forward - Backward Matrix Pencil (TFBMP) is an extension of the Matrix Pencil Method. Total Forward - Backward is the pre-processing technique to break the correlative property of received signals. This fact helps the Matrix Pencil method to estimate the DOA information of coherent incoming signals. Although TFBMP deals with a larger database, however it is more efficient than the original method, especially for a multipath environment. TFBMP was utilized for the high – resolution frequency estimator. In some scenarios, it provides better estimation results than the other methods such as Fourier technique [10].

In Conventional DF systems, there are two common types of the antenna array: Uniform Circular Antenna (UCA) and Uniform Linear Antenna (ULA). In general, with the same number of elements and the same spacing between adjacent radiators, UCA has a smaller size than ULA. Moreover, by using the UCA array, the angle of arrival of incoming signals can be determined from all directions in the azimuth plane. Meanwhile, when using the ULA array, the DOA information is only estimated from -90^o to 90^o. That is why the UCA is of very practical interest and is often adopted in radar and sonar systems as well as in communication system for 3D channel modeling purpose.

In this paper, we develop an approach based on the TFBMP technique to solve the DOA estimation problem using an M – element Uniform Circular Antenna (UCA) array. The performance of this method will be assessed in many cases that depend on the characteristics of incoming signals as well as number of snapshots of data.

The paper is organized as follows. Section II describes the model of the incoming signals based on phase mode excitation beam – forming. In Section III, we present in detail the TFBMP technique for DOA estimation. Simulation results are shown in Section IV. Conclusion is given in Section V.

II. SIGNAL MODEL BASED ON PHASE MODE EXCITATION BEAMFORMING

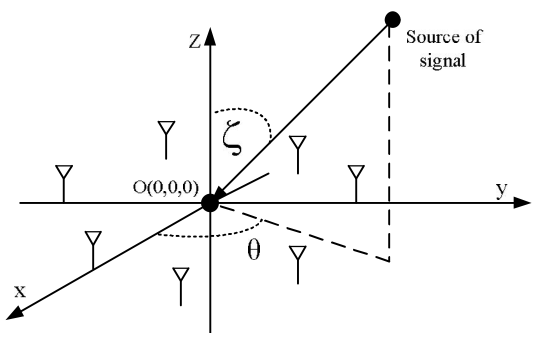

Fig. 1. Antenna array in the coordinate system

In our research, we utilize the Uniform Circular Antenna Array (UCA) model, which includes a set of M isotropic antenna elements spaced around a circle with fixed radius R. We have defined the reference point as the origin of the three - Dimensional – Cartesian – coordinate system shown in Fig.1.

Assuming all signals impinging on the array are narrowband signals. For an M – element array depicted in Fig. 1, let θ be the DOA in azimuth, ζ be the DOA in elevation. The phase difference between any antenna and the reference point is given by

$$\Delta\Psi_m = \beta(x_m cos\theta sin\zeta + y_m sin\theta sin\zeta + z_m cos\zeta), \quad (1)$$

where x_m, y_m, z_m are the three – dimensional – coordinate of each antenna element and $\beta = \frac{2\pi}{\lambda}$ is the propagation factor. Defining the array – manifold vector for an M – element array as

$$a(\theta,\zeta) = [a_0(\theta,\zeta), a_1(\theta,\zeta) \dots a_{M-1}(\theta,\zeta)]^T, \quad (2)$$

where a_m is the phase response at antenna m^{th} relative to the reference point:

$$a_m = g_m e^{-j\Delta\Psi_m}. \quad (3)$$

The single signal received by the array is $y(t) = s(t)a(\theta,\zeta)$, where $s(t)$ is the complex baseband envelope of an incoming signal and g_m is the gain of antenna. Adding receiver noise and allowing K sources, the signal model for the data snapshots at time index is

$$\boldsymbol{Y} = \boldsymbol{\Lambda S} + \mathcal{E}, \quad (4)$$

where

$$\boldsymbol{\Lambda} = [a(\theta_1,\zeta_1), a(\theta_2,\zeta_2), \dots a(\theta_K,\zeta_K)], \quad (5)$$

$$\boldsymbol{S} = [s_1(t), s_2(t), s_3(t) \dots s_K(t)]^T, \quad (6)$$

$\mathcal{E}$ is a noise vector representing AWGN in the received signal path and is assumed to be independent for each antenna and receiver path. In this paper, we only deal with the signal coming on the same plane of the array. Therefore, $\zeta = 90^o$ and $z = 0$. If it is assumed that spatial coordinates of a Uniform Circular Array with M elements are

$$x_m = Rcos(\frac{2\pi m}{M}), \quad (7)$$

$$y_m = Rsin(\frac{2\pi m}{M}), \quad (8)$$

and according to Eq.1, Eq.3, Eq.7 and Eq.8, the single signal collected by m^{th} antenna element is given by

$$y_m(t) = s(t)a_m + \mathcal{E}(t)$$
$$= A(t)e^{-j\left(\frac{2\pi R}{\lambda}\cos\left(\frac{2\pi m}{M}-\theta\right)\right)} + \mathcal{E}(t), \quad (9)$$

where $A(t) = s(t)g_m$ and λ is the carrier wavelength.

In practice, there are several radio signals crossing the antenna array simultaneously. The received signal at each antenna element will be the sum of all arriving radio signals. In case of K signals approaching the array from some azimuth directions $\theta_1, \theta_2 \ldots \theta_k$, the received signal at the m^{th} antenna is

$$y_m(t) = \sum_{k=1}^{K} s_k(t) g_m e^{-j\frac{2\pi R}{\lambda}cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}(t)$$
$$= \sum_{k=1}^{K} A_k(t) e^{-j\frac{2\pi R}{\lambda}cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}(t), \quad (10)$$

where $s_k(t)$ is the complex baseband envelope of an incoming signal at direction θ_k.

This signal can then be sampled with period T_s at discrete time instants nT_s. The discrete sampled form of the output signal at each antenna element is given as

$$y_m(n) = \sum_{k=1}^{K} A_k(n) e^{-j\frac{2\pi R}{\lambda}cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}(n). \quad (11)$$

Each sample of the output signal is defined as one snapshot of data which will be processed to produce the DOA information. It can be simply expressed as

$$y_m = \sum_{k=1}^{K} A_k e^{-j\frac{2\pi R}{\lambda}cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}. \quad (12)$$

There are many ways to estimate the DOA of θ in Eq.11. However, because of the appearing of quantity $\frac{2\pi m}{M}$, we cannot directly use the TFBMP for the DOA estimation similar to [11]. In order to do that, we have to cancel the quantity $\frac{2\pi m}{M}$ by applying the *Beam – forming* to represent Eq.11. Beam – forming is a term used to describe the process of weighting and summing the output of an array of antennas spatial gain pattern on the array. *Phase mode excitation* [12], [13] of a circular array is the specific type of Beam – forming. For the phase mode excitation based Beam – forming, all weights ($w_m(s)$), which are in turn assigned to the output of array elements, have the equal magnitude. Meanwhile, the phase of each weight w_m is assigned in a linear manner based on the angular position of m^{th} array element relative to a reference element, array element 0. The position of a reference element is located in the reception of the plane wave of the UCA depicted in Fig.2.

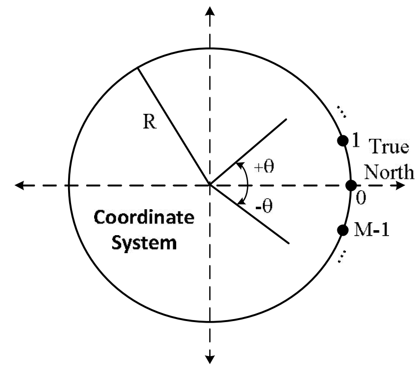

Fig. 2. Reception of plane wave of UCA

In [12], the weight vector that excites the array with phase mode i is mathematically defined as

$$w_i = \frac{1}{M}\left[1 \;\; e^{-\frac{j2\pi i}{M}} \ldots e^{-\frac{j2\pi im}{M}} \ldots e^{-\frac{j2\pi i(M-1)}{M}}\right]^T. \quad (13)$$

By applying the Beam – forming weights to the array - manifold vector of the circular array, the spatial response pattern of the UCA after Beam – forming based on excitation of phase mode i of the array can be approximated by

$$f_i(\theta) = w_i^H a^c(\theta) \approx j^{|i|}\mathcal{J}_{|i|}(\frac{2\pi}{\lambda}R)e^{ji\theta}, \quad (14)$$

where $a^c(\theta)$ is the array – manifold vector with DOA of θ and $\mathcal{J}_i(x)$ is the Bessel function of the first kind of order i. Based on [12], we must also note that the number of modes of a given UCA that can be excited is limited. We call the maximum number of modes N and the selection of an appropriate value for the maximum excited mode N satisfies:

$$N \approx \frac{2\pi}{\lambda}R. \quad (15)$$

Collecting the weights for all modes with $|i| \leq N$, we define the phase mode excitation beam – forming as

$$\widetilde{\boldsymbol{F}}_{pi}^{H} = \boldsymbol{C}\boldsymbol{V}^{H}, \quad (16)$$

where

$$\boldsymbol{C} = diag\{j^{-N}, \ldots, j^{-1}, j^{0}, j^{-1}, \ldots, j^{-N}\}, \quad (17)$$

$$V^{H} = \sqrt{M}[w_{-N} \ldots w_{0} \ldots w_{N}]. \quad (18)$$

After applying the Beam – former to the array – manifold vector of the UCA, the *Beam – space manifold* of the UCA can be defined as

$$\tilde{a}(\theta) = \widetilde{\boldsymbol{F}}_{\boldsymbol{i}}^{H} a^{c}(\theta) = \boldsymbol{C}\boldsymbol{V}^{H} a^{c}(\theta) = \sqrt{M}\boldsymbol{\mathcal{J}}_{\xi} c(\theta), \quad (19)$$

where

$$\boldsymbol{\mathcal{J}}_{\xi} = diag\{\mathcal{J}_{-N}(\xi), \ldots, \mathcal{J}_{0}(\xi), \ldots \mathcal{J}_{N}(\xi)\}, \quad (20)$$

$$\boldsymbol{\mathcal{J}}_{\xi} = diag\left\{\mathcal{J}_{-N}\left(\frac{2\pi}{\lambda}R\right), \ldots \mathcal{J}_{0}\left(\frac{2\pi}{\lambda}R\right), \ldots \mathcal{J}_{N}\left(\frac{2\pi}{\lambda}R\right)\right\}, \quad (21)$$

$$c(\theta) = [e^{-jN\theta}, \ldots, e^{-j\theta}, e^{j0}, e^{j\theta}, \ldots, e^{jN\theta}]^{T}. \quad (22)$$

Based on Eq.19, the output signal from antenna array in Beam – space can be rewritten as

$$\tilde{y}_{m}(n) = \begin{bmatrix} \tilde{y}_{-N}(n) \\ \vdots \\ \tilde{y}_{N}(n) \end{bmatrix} = \widetilde{\boldsymbol{F}}_{i}^{H} y_{m}(n)$$
$$= \sum_{k=1}^{K}\left(\tilde{a}(\theta_{k})A_{k}(n)\right) + \widetilde{\boldsymbol{\mathcal{E}}}(n). \quad (23)$$

According to the Eq.22 and Eq.23, the expression for the observed excitation of single mode i can be expressed as

$$\tilde{y}_{m,i}(n) = \sum_{k=1}^{K}\left(\mathcal{J}_{|i|}\left(\frac{2\pi}{\lambda}R\right)e^{ji\theta_{k}}A_{k}(n)\right) + \tilde{\mathcal{E}}_{m,i}(n), \quad (24)$$

where $-N \leq i \leq N$.

III. DOA ESTIMATION USING TOTAL FORWARD - BACKWARD MATRIX PENCIL METHOD

Assuming that K signals approach the array in presence of white noise, the output signal at each antenna element is presented as in Eq.11. When applying Phase mode excitation Beam – forming, the output signal can be mathematically represented as in Eq.24. Because the Bessel function term $\mathcal{J}_{|i|}\left(\frac{2\pi}{\lambda}R\right)$ does not depend on k index, we can easily cancel this term by multiplying a normalization coefficient $\Delta i = \left(\mathcal{J}_{|i|}\left(\frac{2\pi}{\lambda}R\right)\right)^{-1}$ to each element of the Beam – space data vector. Therefore, we can get

$$\bar{y}_{m,i}(n) = \Delta_{m,i}\tilde{y}_{m,i}(n)$$
$$= \frac{\tilde{y}_{m,i}(n)}{\mathcal{J}_{|i|}(\frac{2\pi}{\lambda}R)} = \sum_{k=1}^{K} A_{k}(n)z_{k}^{i} + \bar{\mathcal{E}}_{m,i}(n), \quad (25)$$

where $z_k = e^{j\theta_k}$ and $-N \leq i \leq N$.

We now estimate the DOA information of the output signal as presented in Eq.25.

We need to set $M' = 2N + 1$ and choose the pencil parameter L with the condition $\frac{M}{3} \leq L \leq \frac{M}{2}$ because of the efficient noise filtering issue described in [5]. Based on TFBMP, we have defined the matrices $\boldsymbol{Y}_{0fb}$ and $\boldsymbol{Y}_{1fb}$ as

$$\boldsymbol{Y}_{0fb_{2(M'-L)\times L}} = \begin{bmatrix} z_0 & z_1 & \cdots & z_{L-2} & z_{L-1} \\ z_L^{*} & z_{L-1}^{*} & \cdots & z_2^{*} & z_1^{*} \end{bmatrix}, \quad (26)$$

$$\boldsymbol{Y}_{1fb_{2(M'-L)\times L}} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{L-1} & z_L \\ z_{L-1}^{*} & z_{L-2}^{*} & \cdots & z_1^{*} & z_0^{*} \end{bmatrix}, \quad (27)$$

where '$*$' denotes complex conjugate and z_τ ($\tau = 0, \ldots, L$) is defined as

$$z_{\tau}^{T} = [y_{\tau}\, y_{\tau+1} \ldots y_{M'-L+\tau-1}],$$
$$(\tau = 0, \ldots, L). \quad (28)$$

Based on Eq.26 and Eq.27, the Matrix Pencil, $\boldsymbol{Y}_{1fb} - z\boldsymbol{Y}_{0fb}$ (z is complex scalar), can be built to DOA estimate, but, for noisy data, the best way is to perform a Singular Value Decomposition (SVD) on the "all data" matrix. This matrix is given by

$$\boldsymbol{Y}_{fb_{2(M'-L)\times(L+1)}} = \begin{bmatrix} z_0 & z_1 & \cdots & z_{L-1} & z_L \\ z_L^* & z_{L-1}^* & \cdots & z_1^* & z_0^* \end{bmatrix}. \quad (29)$$

It is seen that $\boldsymbol{Y}_{0fb}$ and $\boldsymbol{Y}_{1fb}$ are obtained from $\boldsymbol{Y}_{fb}$ by deleting, respectively, its $(L+1)^{th}$ and first columns

$$\boldsymbol{Y}_{fb_{2(M'-L)\times(L+1)}} = [\boldsymbol{Y}_{0fb_{2(M'-L)\times L}}, c_{L+1}], \quad (30)$$

$$\boldsymbol{Y}_{fb_{2(M'-L)\times(L+1)}} = [c_1, \ \boldsymbol{Y}_{1fb_{2(M'-L)\times L}}]. \quad (31)$$

On the other hand, the SVD of $\boldsymbol{Y}_{fb}$ is

$$\boldsymbol{Y}_{fb_{2(M'-L)\times(L+1)}} = \boldsymbol{U}_{2(M'-L)\times 2(M'-L)}\boldsymbol{\Sigma}_{2(M'-L)\times(L+1)}\boldsymbol{V}^H_{(L+1)\times(L+1)}, \quad (32)$$

where H denotes complex conjugate transpose of a matrix, $\boldsymbol{U}, \boldsymbol{\Sigma}$, and $\boldsymbol{V}$ are given by

$$\boldsymbol{\Sigma} = diag\{\sigma_1, \sigma_2 \dots \sigma_p\}, \quad (33)$$

where $p = min\{2(M'-L), L+1\}$ and $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_p \geq 0$,

$$\boldsymbol{U} = \left[u_1, u_2, \dots, u_{2(M'-L)}\right], \quad (34)$$

$$\boldsymbol{Y}^H_{fb} u_k = \sigma_k v_k, \ (k = 1, \dots, p) \quad (35)$$

$$\boldsymbol{V} = \left[v_1, v_2, \dots, v_{(L+1)}\right], \quad (36)$$

$$\boldsymbol{Y}^H_{fb} v_k = \sigma_k u_k, \ (k = 1, \dots, p) \quad (37)$$

$$\boldsymbol{U}^H\boldsymbol{U} = \boldsymbol{I}, \boldsymbol{V}^H\boldsymbol{V} = \boldsymbol{I}, \quad (38)$$

where σ_k are the singular values of $\boldsymbol{Y}_{fb}$ and the vector u_k and v_k are the k^{th} left and right singular vector, respectively.

The problem can be computationally improved by applying the singular value filtering to obtain the K largest singular values of $\boldsymbol{Y}_{fb}$.

$$\overline{\boldsymbol{Y}}_{fb_{2(M'-L)\times(L+1)}} = \overline{\boldsymbol{U}}_{2(M'-L)\times K}\overline{\boldsymbol{\Sigma}}_{K\times K}\overline{\boldsymbol{V}}^H_{K\times(L+1)}, \quad (39)$$

where

$$\overline{\boldsymbol{\Sigma}} = diag\{\sigma_1, \sigma_2 \dots \sigma_K\} \quad (40)$$

has the K largest singular values of $\boldsymbol{\Sigma}$, and the matrices $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{V}}$ are formed by extracting the singular vectors corresponding to K singular values. Extract $\overline{\boldsymbol{V}_0}$ and $\overline{\boldsymbol{V}_1}$ from $\overline{\boldsymbol{V}}$ as follows:

$$\overline{\boldsymbol{V}} = [\overline{\boldsymbol{V}}_{\boldsymbol{0}}, v_{L+1}], \overline{\boldsymbol{V}} = [v_1, \overline{\boldsymbol{V}}_1]. \quad (41)$$

Same as above, $\overline{\boldsymbol{Y}}_{0fb}$ and $\overline{\boldsymbol{Y}}_{1fb}$ are established as

$$\overline{\boldsymbol{Y}}_{0fb} = \overline{\boldsymbol{U}}\overline{\boldsymbol{\Sigma}}\overline{\boldsymbol{V}}^H_{\boldsymbol{0}}, \quad (42)$$

$$\overline{\boldsymbol{Y}}_{1fb} = \overline{\boldsymbol{U}}\overline{\boldsymbol{\Sigma}}\overline{\boldsymbol{V}}^H_1. \quad (43)$$

Now, consider the matrix pencil

$$\overline{\boldsymbol{Y}}_{1fb} - z\overline{\boldsymbol{Y}}_{0fb} \quad (44)$$

and left multiplying Eq.44 by $\overline{\boldsymbol{Y}}^+_{0fb}$ yields

$$q^H(\overline{\boldsymbol{Y}}_{1fb}\overline{\boldsymbol{Y}}^+_{0fb} - z\boldsymbol{I}) = 0^H, \quad (45)$$

where $\overline{\boldsymbol{Y}}^+_{0fb}$ is the Moore – Penrose pseudo – inverse of $\boldsymbol{Y}_{0fb}$,

$$\overline{\boldsymbol{Y}}^+_{0fb} = (\overline{\boldsymbol{V}}^H_0)^+\overline{\boldsymbol{\Sigma}}^{-1}\overline{\boldsymbol{U}}^+. \quad (46)$$

Substituting Eq.43 and Eq.46 into Eq.45 the equivalent generalized eigen – problem becomes

$$q^H(\overline{\boldsymbol{V}}^H_1 - z\overline{\boldsymbol{V}}^H_0) = 0^H. \quad (47)$$

Left multiplying Eq.47 by $\overline{\boldsymbol{V}_0}$, we have

$$q^H(\overline{\boldsymbol{V}}^H_1\overline{\boldsymbol{V}}_0 - z\overline{\boldsymbol{V}}^H_0\overline{\boldsymbol{V}}_0) = 0^H. \quad (48)$$

Using the values of the generalized eigenvalues, z, of Eq.48, angles of arrival can be estimated as

$$\theta_k = \Im(ln(z_k)), \quad (49)$$

Where $\Im(ln(z_k))$ is the imaginary part of $ln(z_k)$.

IV. SIMULATION RESULTS

The DF algorithm is simulated using Matlab in order to examine the performance of the algorithm. In this paper, we assume three signals ($K = 3$) impinging on a 10 – element antenna array ($M = 10$) with inter - spacing $d = 0.3\lambda$. The value of d parameter is optimizedin order to reduce the size of the antenna array as well as to ensure the acceptable mutual coupling factor between antenna elements. The simulation is performed in two cases: incoherent and coherent signals.

A. Incoherent signals

In this paper, three incoherent signals at 1 GHz, 1.2 GHz and 1.3GHz are supposed to impinge on the antenna array at the DOA of -50^o, 60^o and 160^o, respectively. One of the most significant advantages of TFBMP is that it can extract the DOA information with only one snapshot [14]. The estimated DOAs in the simulation are the numerical values as in Eq.49. However, in order to demonstrate visually the results, we illustrate the DOA in XOY plane, in which the X – Axis is the DOA of incoming signals and the Y – Axis is the indicating factor. This factor is set to 1 corresponding to the estimated DOA in X - Axis. Figure 3 presents the DOA estimation of those signals in the AWGN channel in which the SNR is set to 10dB. We can see that the DOA of signal of interest is estimated accurately by the proposed algorithm with very small error. In Fig. 4, the simulation is performed in the AWGN channel with the variable SNRs from −10dB to 40dB. The RMSE is presented to prove the accuracy of this algorithm. It is easy to see that this method is quite good in a white noise environment although with only one snapshot. In similar situation, the DOA information could not be estimated by the Root – Music algorithm as shown in Fig.5. The fact is due to the inaccurate estimate of the correlation matrix.

In order to evaluate the influence of the number of snapshots on the performance of this algorithm, we execute this method with the same inputs as above with 1000 snapshots. The simulation result shown in Fig.6 indicates that when we increase the number of snapshots, the accuracy of this method increases with RMSE equal to 0.3379^o in comparison with 0.5307^o when one snapshot captured. Moreover, with multi – snapshots, this algorithm in the AWGN channel with varying SNRs operates more stably than in case of one snapshot as shown in Fig.7 . Obviously, we can get the better results when we increase the number of snapshots. The simulation result plotted in Fig. 8 has statement proved that. However, it can be observed that when the number of snapshots is more than 90, the accuracy of the algorithm seems to be invariable. On the other hand, when the number of snapshot is increased, the computation time also significantly increases. Therefore, the trade - off between the computation time and the accuracy of the algorithm should be taken into account.

Furthermore, the angle resolution of this method is also a factor to be assessed. In order to do that we perform this algorithm with some pairs of incoherent incident signals whose SNRs are set to 10dB. The simulation results are stored in Table 1. If we consider the RMSE of desired results less than 2 degrees, from Table 1, we can easily see that for the case of incoherent incoming signals, this method will work well with the resolution approximately 5 degrees.

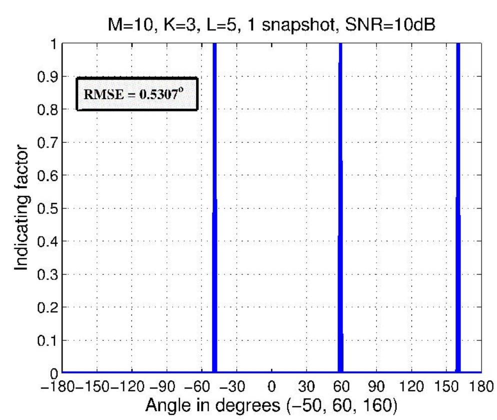

Fig.3. DOA estimation results of three incoherent signals at the DOA of -50^o, 60^o and 160^o with one snapshot.

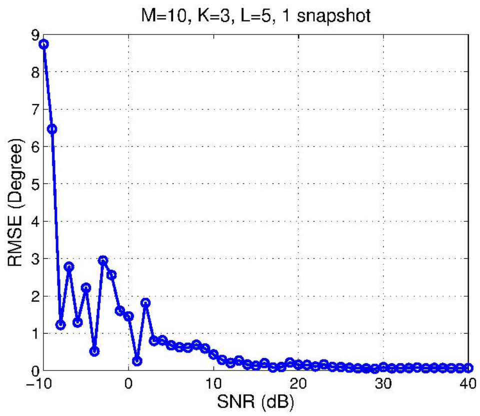

Fig.4. Estimation accuracy at the DOA of -50°, 60° and 160° of three incoherent signals with one snapshot

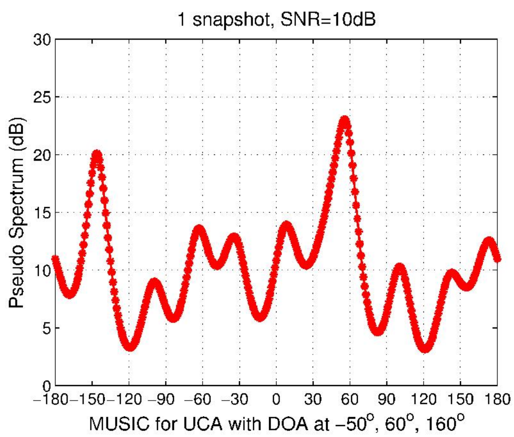

Fig.5. DOA estimation using Root - MUSIC with one snapshot

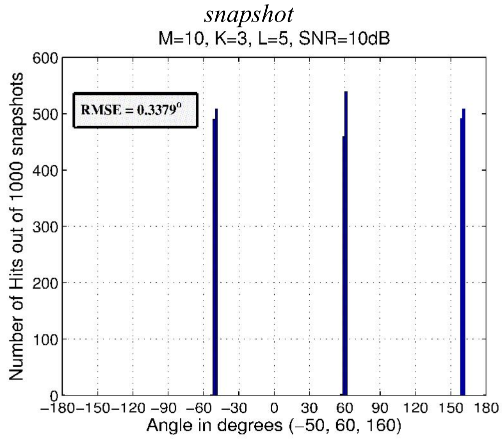

Fig.6. 1000 snapshots simulation results of three incoherent signals at the DOA of -50°, 60° and 160°

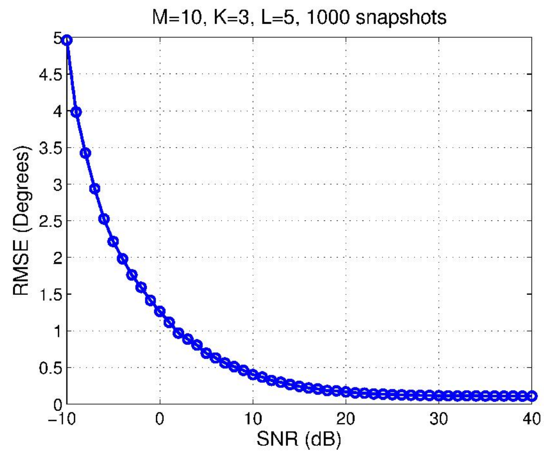

Fig. 7. Estimation accuracy at the DOA of -50°, 60° and 160° of three incoherent signals with 1000 snapshots

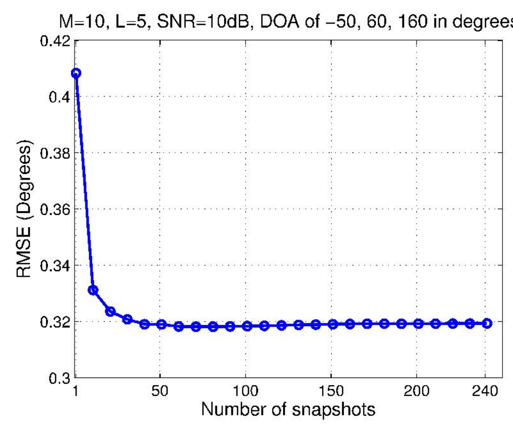

Fig.8. DOA estimation with the different number of snapshots

Table 1. Angle resolution for incoherent signals

θ_1	θ_2	DOA estimation results		RMSE
30°	31°	30.5955°	96.9867°	46.6615°
30°	32°	31.0707°	43.0304°	7.8363°
30°	33°	31.7978°	42.0791°	6.5446°
30°	34°	32.1605°	36.9469°	2.5838°
30°	35°	31.1450°	34.2202°	0.9796°
30°	36°	30.7914°	35.2331°	0.7793°
30°	37°	30.3921°	36.2355°	0.6075°
30°	38°	30.3557°	37.1282°	0.6658°

B. Coherent signals

Another remarkable advantage of TFBMP is that it can extract the DOA information of coherent incoming signals in highly correlative environment

[15], which could not be found by the Root – MUSIC algorithm as shown in Fig.9. In this case, the algorithm has to break the correlation between the signals and then estimates and produces the DOA information. In order to assess performance of the algorithm, we execute this algorithm in similar situations with the case of incoherent signals. Figure 10 demonstrates the simulation results of three coherent signals at 1 GHz, which come from the DOA of -30^o, 0^o and 110^o with one snapshot.

Figure 11 shows the accuracy of DOA estimation according to the variable SNRs. And the relation between the accuracy of the algorithm and number of snapshots is shown in Fig.12. We can see that the DOA information of the coherent incoming signals can still be exactly calculated by this algorithm. However, the performance of the algorithm is less than the incoherent case. In this case, the RMSE increases and we need at least 150 snapshots, the accuracy could be invariable. This phenomenon is due to the characteristic of correlation signals.

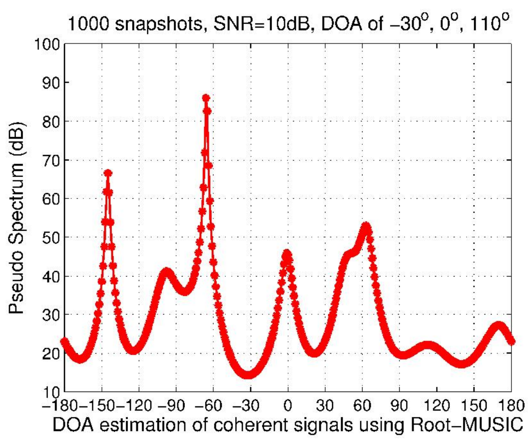

Fig.9. DOA estimation of three coherent signals in highly correlative environment at 1GHz from -30^o, 0^o and 110^o using Root – MUSIC

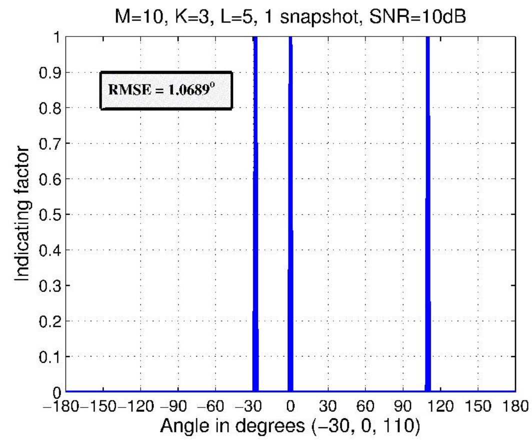

Fig.10. DOA estimation of three coherent signals at 1GHz

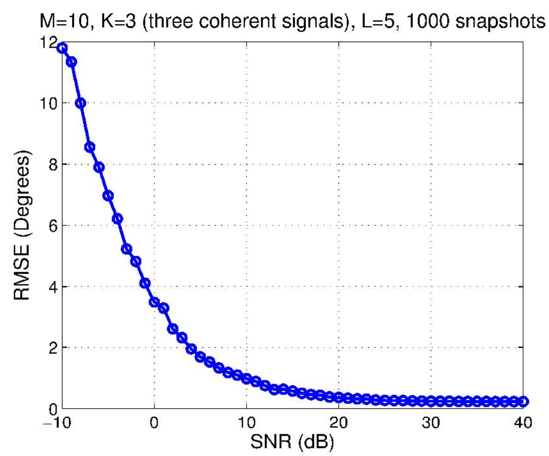

Fig.11. Estimation accuracy at the DOA of -30^o, 0^o and 110^o of three coherent signals with 1000 snapshots

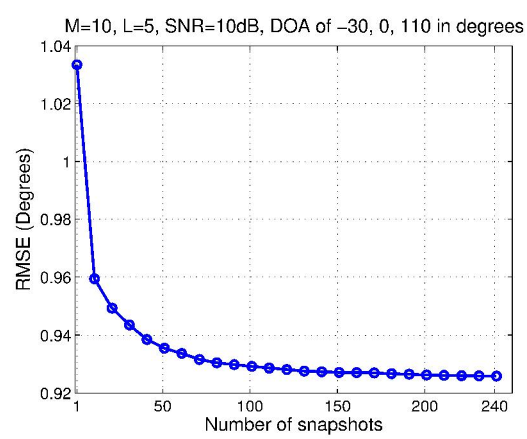

Fig.12. DOA estimation with the different number of snapshots (in case of coherent signals)

Moreover, the angle resolution is the factor to compare the performace of the algorithm between two cases. In order to do that we perform this algorithm with the same value inputs of DOA as in the incoherent case. The simulation results are stored in Table 2. If we still consider the RMSE of desired results less than 2 degrees, from Table 2, we can easily see that this method will work well with the resolution of approximately 8 degrees. Evidently, the angle resolution of this method in this case decreases.

Table 2. Angle resolution for coherent signals

θ_1	θ_2	DOA estimation results		RMSE
30°	31°	30.5184°	-7.0974°	26.2341°
30°	32°	30.7883°	5.8804°	17.0766°
30°	33°	31.7590°	-31.4549°	43.4640°
30°	34°	31.4443°	61.6602°	19.5854°
30°	35°	32.0660°	39.7760°	3.6795°
30°	36°	31.2046°	39.7031°	2.7535°
30°	37°	31.8203°	40.0928°	2.5376°
30°	38°	30.8867°	39.4569°	1.2060°
30°	39°	28.6920°	38.2658°	1.0606°
30°	40°	28.8908°	39.6352°	0.9038°

V. CONCLUSIONS

We propose, in this paper, an efficient approach for DOA estimation by using Total Forward – Backward Matrix Pencil algorithm. The benefit of this approach is that it can estimate correctly the DOA of coherent incoming signals because it deals directly with the input samples of signal and does not require the estimation of a correlation matrix to produce DOA information. Moreover, this algorithm can operate with only one discrete time sample of signal, which means that the sampling frequency can be considerably reduced. In this way, the computational complexity significantly decreases in comparison with the other algorithms. Obviously, this method is a high - performance DOA estimation algorithm. It is one of the practical ways to estimate the DOA information of the radio frequency signals in the presence of noise.

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