

# All-Optical Modified Discrete Cosine Transform (MDCT) Using MMI Structures

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**Abstract**—The Modified Discrete Cosine Transform (MDCT) have found applications in digital signal, image and video processing and particularly in transform coding systems for data compression and decompression. Multimode interference (MMI) in optical silicon on insulator (SOI) waveguides is attractive for realizing all-optical MDCT transforms as they have the advantages of low loss, ultra-compact size and excellent fabrication tolerances. In this paper, for the first time, a novel approach to realize all-optical MDCT based on multimode interference structures on silicon on insulator platform is proposed. Based on the transfer matrix method, the analytical expressions describing the characteristics of the MMI structures are derived. Designs of the proposed devices are then verified and optimized using 2D and 3D BPM (Beam propagation method) simulations.

**Keywords:** All-optical modified discrete cosine transform (MDCT), image processing, multimode interference coupler, optical signal processing, signal processing

## I. INTRODUCTION

For many years, optical techniques have been considered for a variety of signal processing tasks such as pattern recognition, the generation of ambiguity surfaces for radar signal processing and image processing applications [1,2]. The major reason for using an optical signal processor is its high bandwidth advantage over electronic processors. Due to its high throughput, the application of optical signal processing in optical communication systems is a very attractive research area. Photonic signal processing transforms such as the discrete Fourier transform (DFT), discrete cosine transforms (DCT) and discrete wavelet transforms (DWT) are useful for spatial signal

processing and optical computing such as spectrum analysis, filtering, and encoding, etc.

Early efforts used lens systems [3, 4], directional couplers [5, 6], and single mode star networks [7] to develop optical signal processing transforms such as the Hadamard transform, the DFT and wavelet filters. However, the systems based on these technologies are usually quite large, lack accuracy and require high precision mechanical placement. In addition, the structure for implementing the transforms based on fibre technology requires bulky crossovers of fibre cables. Recently, the design of DFT and DCT transforms using fibre directional couplers has been presented by Moreolo and Cincotti [8]. In the literature [9, 10], a few transforms such as Hadamard transforms and discrete unitary transformations have employed MMI structures and multimode waveguide holograms. However, these devices were designed for the InP material system. For the device using holograms, a complex fabrication process is required. The presence of holograms within the multimode waveguide tends to introduce additional losses.

Recently, a method for realizing all-optical Fourier transform based on multimode interference has been reported [11]. The design of these devices has been implemented on the silica material system. In addition, all-optical discrete cosine transforms and discrete sine transform using multimode interference couplers have been realized [12]. However, these structures still need to use fiber optic directional couplers.

In this paper, we propose a new method to realize all-optical modified discrete cosine transform

(MDCT) based on multimode interference (MMI) structures using silicon waveguides.

MMI devices use the principle of self-imaging within multimode optical waveguides to produce single or multiple images, of an input field, at periodic distances along the waveguide. Such multimode interference couplers have the desirable advantages of low loss, compactness and good fabrication tolerances [13]. In addition, the available material systems used for such multimode devices include polymers, silica on silicon and silicon-on-insulator (SOI). The high-index contrast silicon-on-insulator (SOI) platform has attracted much interest due to its potential for miniaturization, improved performance, and compatibility with existing CMOS technology [14].

The photonic circuits are analyzed and optimized using the transfer matrix method and the beam propagation method (BPM) [15]. A description of the general theory behind the use of multimode structures to achieve the MDCT device presented in Section II. Simulation results of MMI based structures for components in the device structure are covered in Section III. A brief summary of the results of this research is given in Section IV.

## II. GENERAL THEORY

The conventional MMI coupler has a structure consisting of a homogeneous planar multimode waveguide region connected to a number of single mode access waveguides. Figure 1 shows a structure of a rectangular  $N \times N$  MMI coupler, where  $N=3$ ,  $W_{MMI}$  and  $L_{MMI}$  is the width and length of the MMI coupler. The MMI region is sufficiently wide to support a large number of lateral modes (in the  $y$  direction).

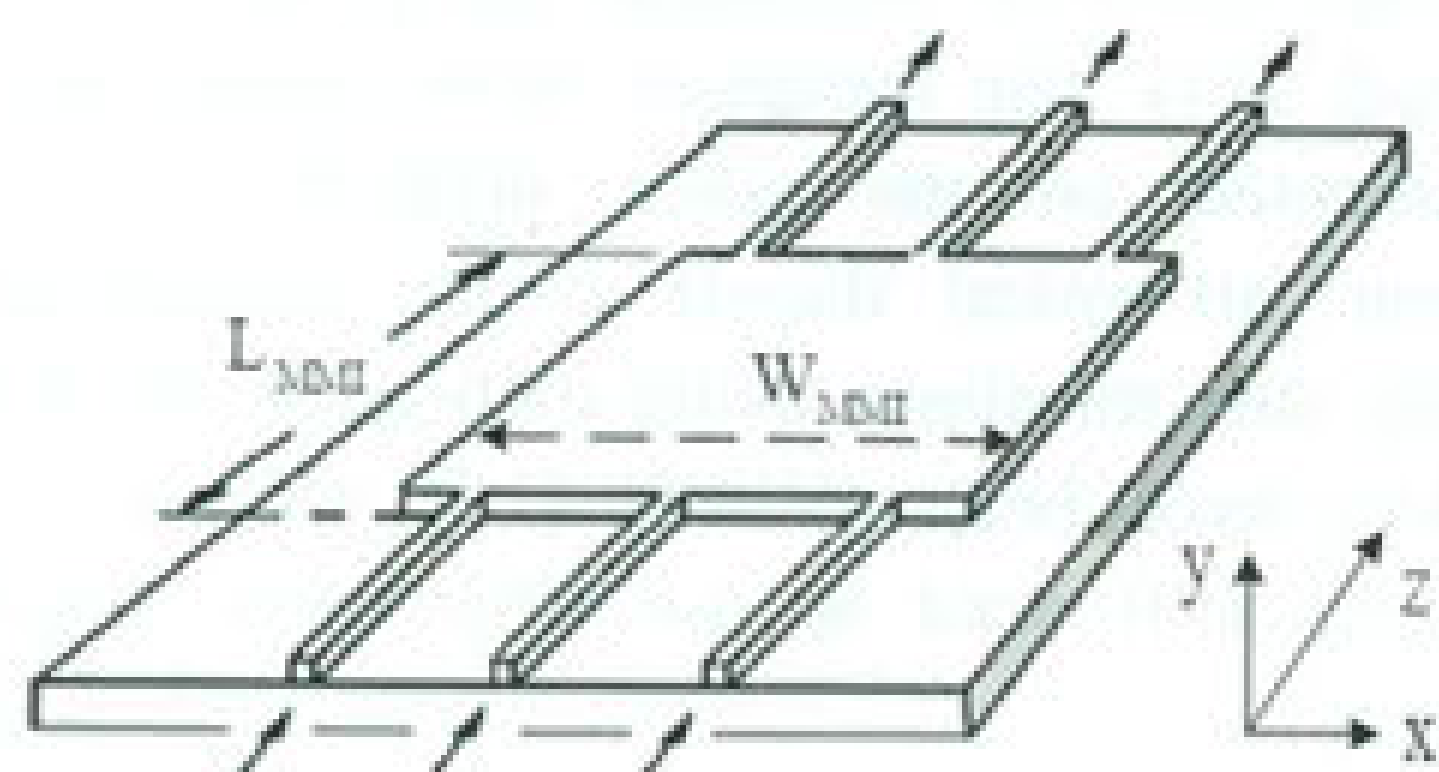


Figure 1. The structure of a 3x3 MMI coupler

The operation of optical MMI coupler is based on the self-imaging principle [13, 14]. Self-imaging is a property of a multimode waveguide by which an input field is reproduced in single or multiple images at periodic intervals along the propagation direction of the waveguide. The central structure of the MMI filter is formed by a waveguide designed to support a large number of modes.

In this paper, the access waveguides are identical single mode waveguides with width  $W_a$ . The input and output waveguides are located at

$$x = (i + \frac{1}{2}) \frac{W_{MMI}}{N}, \quad (i=0, 2, \dots, N-1) \quad (1)$$

The electrical field inside the MMI coupler can be expressed by [16]

$$E(x, z) = \exp(-jkz) \sum_{m=1}^M E_m \exp(j \frac{m^2 \pi}{4\Lambda} z) \sin(\frac{m\pi}{W_{MMI}} x) \quad (2)$$

where  $k = 2\pi n / \lambda$ ,  $\lambda$  is the operating wavelength,  $n$  is the waveguide refractive index and  $M$  is the total number of guided modes in the MMI coupler,  $E_m$  is the summation coefficients. We have the orthogonal set relating the internal modes field to the outer input-output field

$$V_{ir} = \begin{cases} \sqrt{\frac{2}{N}} \sin(\frac{r\pi}{N}(i + \frac{1}{2})) & (r \neq N) \\ \sqrt{\frac{1}{N}} \sin(\pi(i + \frac{1}{2})) & (r = N) \end{cases} \quad (3)$$

Where  $V_{ir}$  is the element on row  $i$  and column  $r$  of a matrix  $V_N$ , which relates the propagation modes inside the waveguide to the output field.

It is assumed that the length of the MMI coupler is set to  $L_{MMI} = 2\Lambda / N$ , where  $\Lambda = nW_{MMI}^2 / \lambda$ . If the common phase term in equation (2) is not considered, the  $i$ th propagating modes will experience different phase shift of  $i^2 \pi / (2N)$  and the matrix  $V_N^T$  is then multiplied by a diagonal matrix with the diagonal elements

$$b_{rr} = \exp(j \frac{r^2 \pi}{2N}) \quad (4)$$

The total transfer matrix of the waveguide from input to the output ports now can be calculated by

$$M = VBVT^T \quad (5)$$

This equation can be rewritten by

$$M_{uv} = je^{j\frac{\pi}{4}} \sqrt{\frac{2}{N}} \sin\left(\frac{\pi(u+\frac{1}{2})(v+\frac{1}{2})}{N}\right) e^{(-j\pi\frac{(u+\frac{1}{2})^2+(v+\frac{1}{2})^2}{2N})} \quad (6)$$

If the phase shifters are added to the input ports and output ports of the MMI structure, the total transfer matrix can be calculated by

$$T = D_{out}MD_{in} \quad (7)$$

Where  $D_{in}$  and  $D_{out}$  are the matrices indicating the contribution of the input and output phase shifters arrays. If the phase shifter are set to be  $\pi(i+\frac{1}{2})^2 / (2N)$ ,  $i=0, 2, \dots, N-1$ , at the input and output waveguides, the total transfer matrix  $T$  can be computed by

$$T_{uv} = je^{j\frac{\pi}{4}} \sqrt{\frac{2}{N}} \sin\left(\frac{\pi(u+\frac{1}{2})(v+\frac{1}{2})}{N}\right) \quad (8)$$

In addition, the DST-IV can be described by the matrix

$$M_{DST} = \sqrt{\frac{2}{N}} \sin\left(\frac{\pi(u+\frac{1}{2})(v+\frac{1}{2})}{N}\right) \quad (9)$$

Therefore, the matrix of equation (8) is the matrix of the DST-IV if the phase factor  $j\exp(j\frac{\pi}{4})$  is neglected.

Also, the matrix of the DCT-IV can be expressed by

$$M_{DCT} = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi(u+\frac{1}{2})(v+\frac{1}{2})}{N}\right) \quad (10)$$

It is can be proved that  $M_{DCT} = PM_{DST}Q$ , where

$$P = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & & \dots & \\ & 1 & & \\ 1 & & & \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

Therefore, the DCT-IV can be implemented using the DST-IV device based on MMI structures by putting a phase shifter  $\pi$  at the even input ports and re-labeling all the output ports with the inverse order.

The MDCT of an input sequence  $\{x_n\}_{n=0, \dots, 2N-1}$  is given by

$$y_k = \sum_{n=0}^{2N-1} x_n \cos\left(\frac{\pi}{N}\left(n+\frac{1}{2}+\frac{N}{2}\right)\left(k+\frac{1}{2}\right)\right) \quad (11)$$

Where  $k=0, 1, \dots, N-1$ . In this study, we propose a new transform called, the modified discrete sine transform or MDST. The MDST of an input sequence

$\{x_n\}_{n=0, \dots, 2N-1}$  is given by

$$y'_k = \sum_{n=0}^{2N-1} x_n \sin\left(\frac{\pi}{N}\left(n+\frac{1}{2}+\frac{N}{2}\right)\left(k+\frac{1}{2}\right)\right) \quad (12)$$

We can rewrite the factor within the cosine and sine functions by

$$\frac{\pi}{N}\left(n+\frac{1}{2}+\frac{N}{2}\right)\left(k+\frac{1}{2}\right) = \frac{\pi}{N}\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right) + \frac{\pi}{2}\left(k+\frac{1}{2}\right) \quad (13)$$

As a result, the MDCT and MDST transforms can be achieved simultaneously by using a general structure as shown in Fig. 2. As an example, we only show the case for 8 input and 4 output signals.

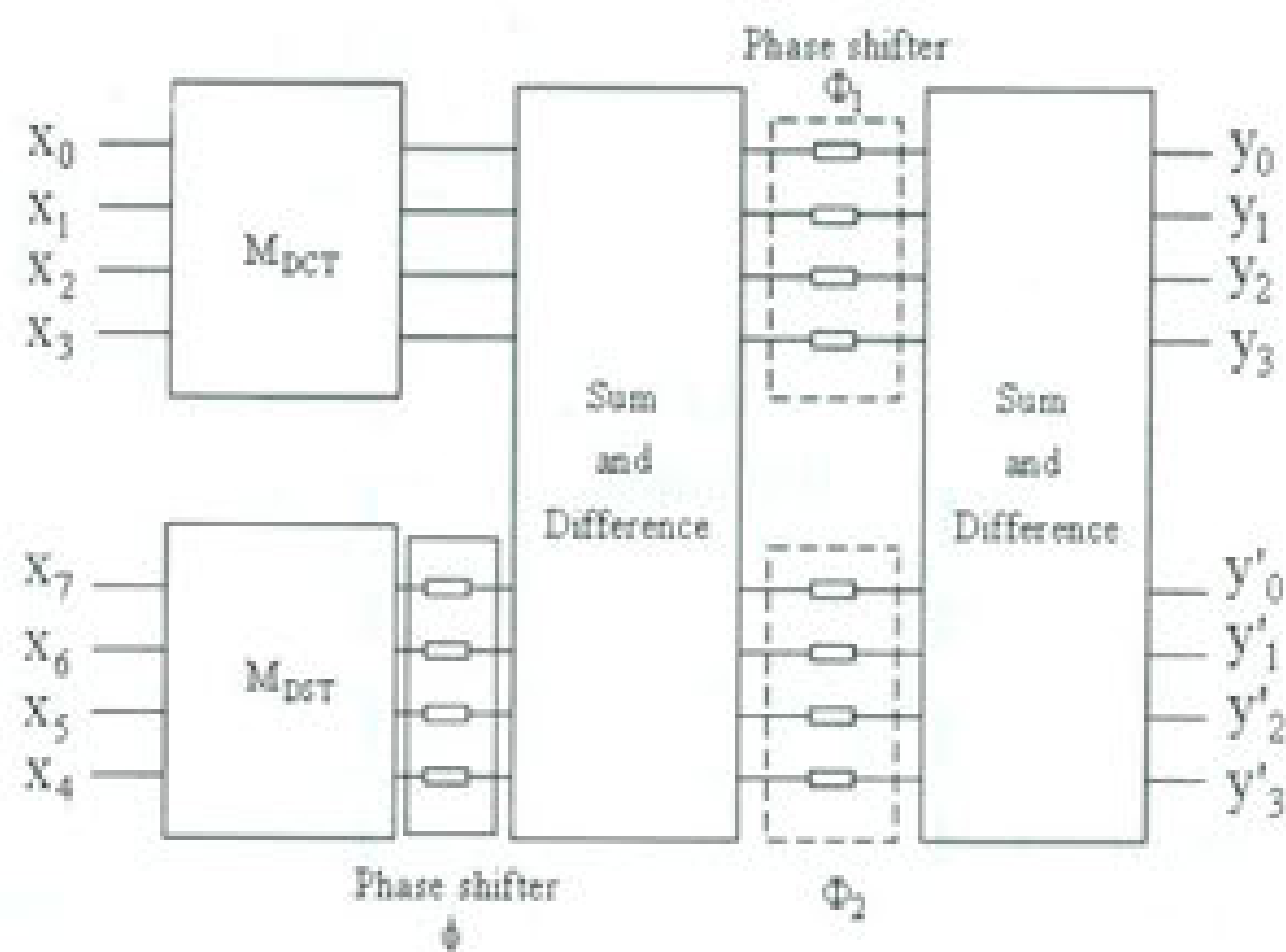


Figure 2. The structure of the MDCT and MDST transforms

Where the matrix of the phase shifters  $\phi$  must be

$$\phi = \begin{pmatrix} \exp(j\frac{\pi}{2}) & 0 & 0 & 0 \\ 0 & \exp(j\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & \exp(j\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & \exp(j\frac{\pi}{2}) \end{pmatrix} \quad (14)$$

and one can prove that the phase shifters at the output of the sum and difference unit in Fig. 2 can be expressed by

$$\Phi_1 = \begin{pmatrix} e^{j\frac{\pi}{2}(\frac{1}{2})} & 0 & 0 & 0 \\ 0 & e^{j\frac{\pi}{2}(1+\frac{1}{2})} & 0 & 0 \\ 0 & 0 & e^{j\frac{\pi}{2}(2+\frac{1}{2})} & 0 \\ 0 & 0 & 0 & e^{j\frac{\pi}{2}(3+\frac{1}{2})} \end{pmatrix} \quad (15)$$

$$\Phi_2 = \begin{pmatrix} e^{-j\frac{\pi}{2}(\frac{1}{2})} & 0 & 0 & 0 \\ 0 & e^{-j\frac{\pi}{2}(1+\frac{1}{2})} & 0 & 0 \\ 0 & 0 & e^{-j\frac{\pi}{2}(2+\frac{1}{2})} & 0 \\ 0 & 0 & 0 & e^{-j\frac{\pi}{2}(3+\frac{1}{2})} \end{pmatrix} \quad (16)$$

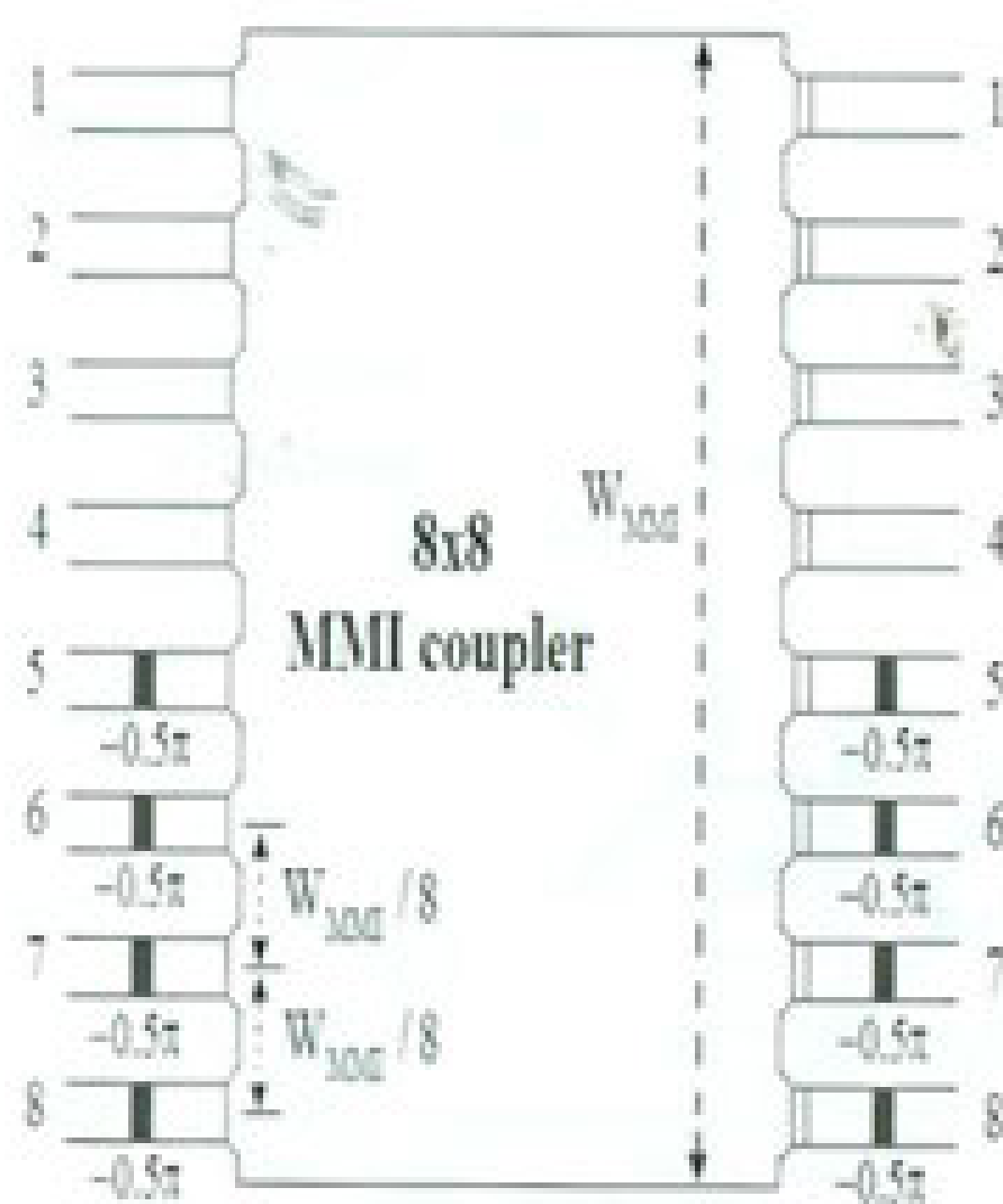


Figure 3. Sum and difference unit based on an 8x8 MMI coupler

The sum and difference unit performs the sum and difference of the input signals. In Fig.2, the signals at four upper arms are sum of input signals and the signals at four lower arms are difference of input signals. This sum and difference unit can be realized by using 8x8 MMI structures with phase shifts located at the input and output ports. The implementation of this device is shown in Fig. 3.

Therefore, the MDCT and MDST can be achieved by using cascading MMI structures as shown in Fig. 2.

### III. SIMULATION RESULTS AND DISCUSSION

The waveguide structure used in the designs is shown in Fig. 4. Here, SiO<sub>2</sub> ( $n_{\text{SiO}_2} = 1.46$ ) is used as the upper cladding material. An upper cladding region is needed for devices using the thermo-optic effect in order to reduce loss due to metal electrodes. Also, the upper cladding region is used to avoid the influence of moisture and environmental temperature [17].

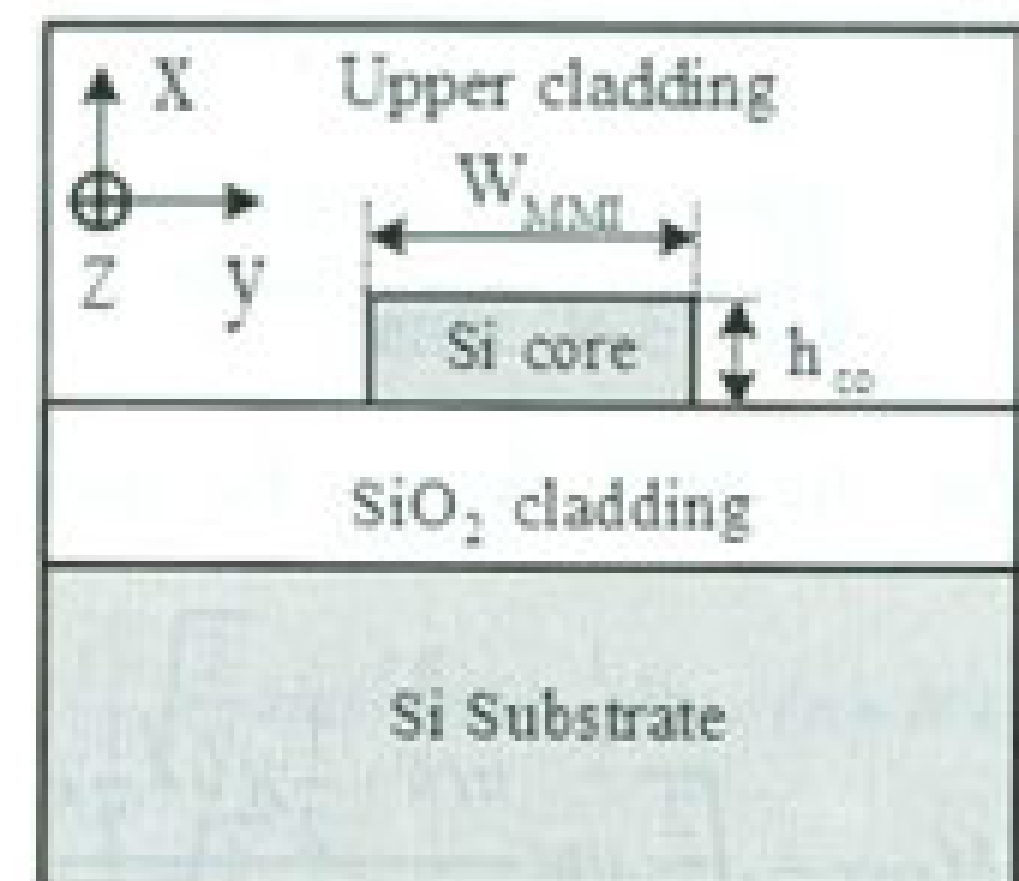
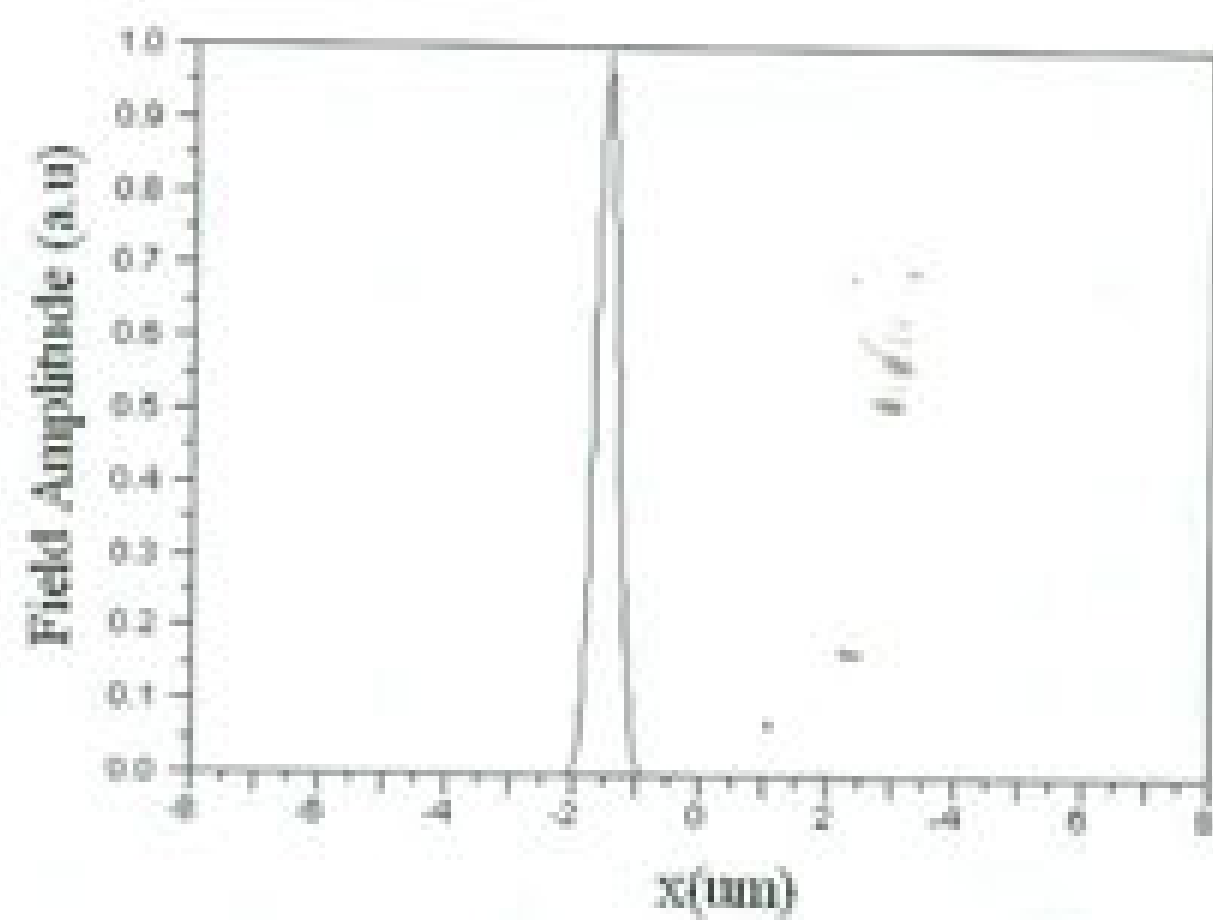


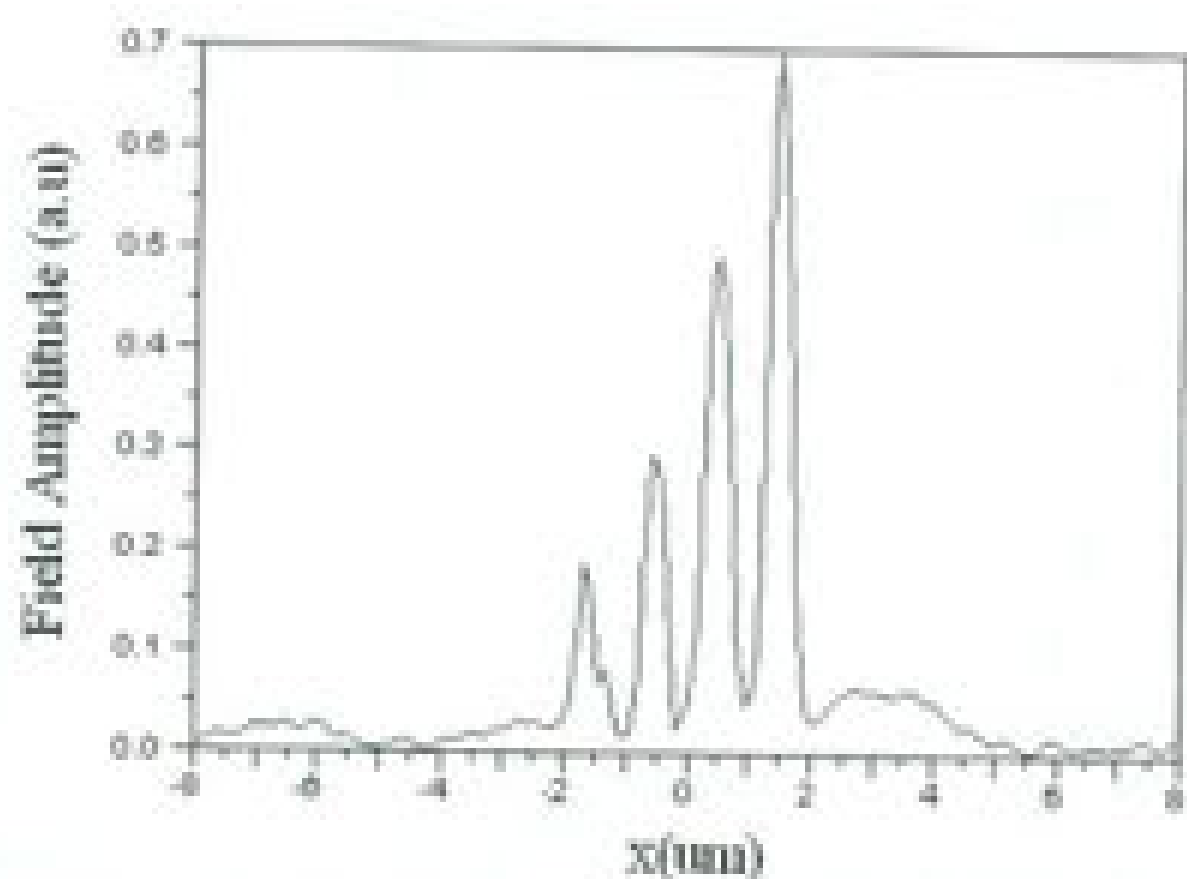
Figure 4. Silicon waveguide cross-section used in the designs of the proposed device

The parameters used in the designs are as follows: the waveguide has a standard silicon thickness of  $h_{\text{co}} = 220\text{nm}$  and access waveguide widths are  $W_a = 0.5\ \mu\text{m}$  for single mode operation. It is assumed that the designs are for the TE polarization at a central optical wavelength  $\lambda = 1550\text{nm}$ .

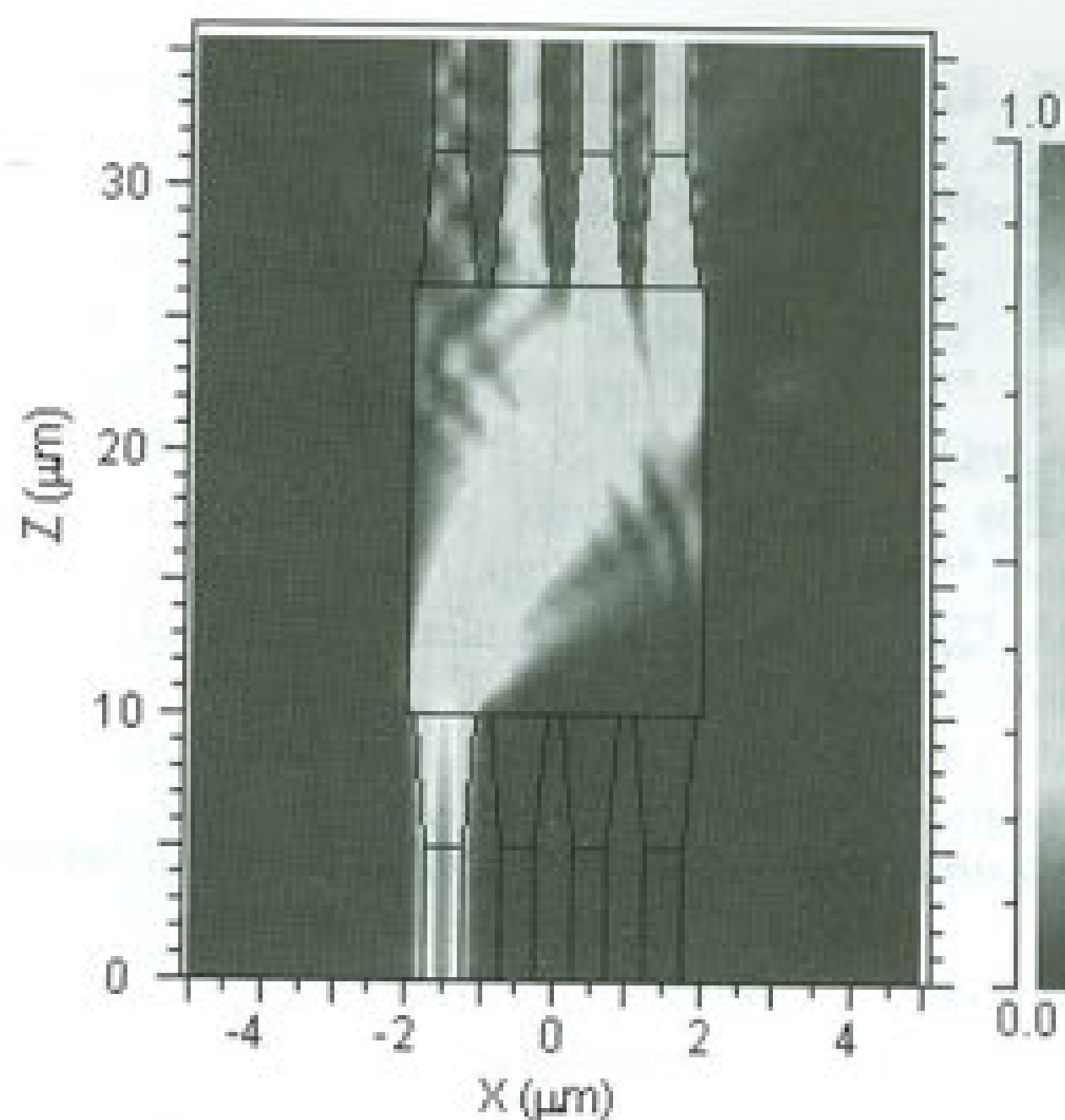
The width of the MMI is  $W_{\text{MMI}} = 4\ \mu\text{m}$  for 4 point DCT-IV and DST-IV. The access waveguide is tapered to a width of  $W_{\text{tp}} = 800\text{nm}$  to improve device performance [18]. The optimized length of the MMI coupler calculated by using 3D-BPM is  $L_{\text{MMI}} = 16.2\ \mu\text{m}$ . As an example, we assume that the



(a) Field amplitude at the input ports for input vector  $(1000)^T$



(b) Field amplitude at the output ports for input vector  $(1000)^T$  of the 4 point DCT using  $4 \times 4$  MMI structures



(c) Field propagation by using the BPM simulation when the input vector is  $(1000)^T$  and the all-optical DCT-IV is performed

Figure 5. BPM simulation result of the DCT-IV transform: (a) input amplitude, (b) output amplitude and (c) field propagation

input vector is to be  $(x_0 x_1 x_2 x_3)^T = (1000)^T$  and the all-optical DCT-IV is performed. The normalized input and output amplitudes are shown in Fig. 5(a) and 5(b), respectively. The 3D BPM simulation for this case is shown in Fig. 5(c).

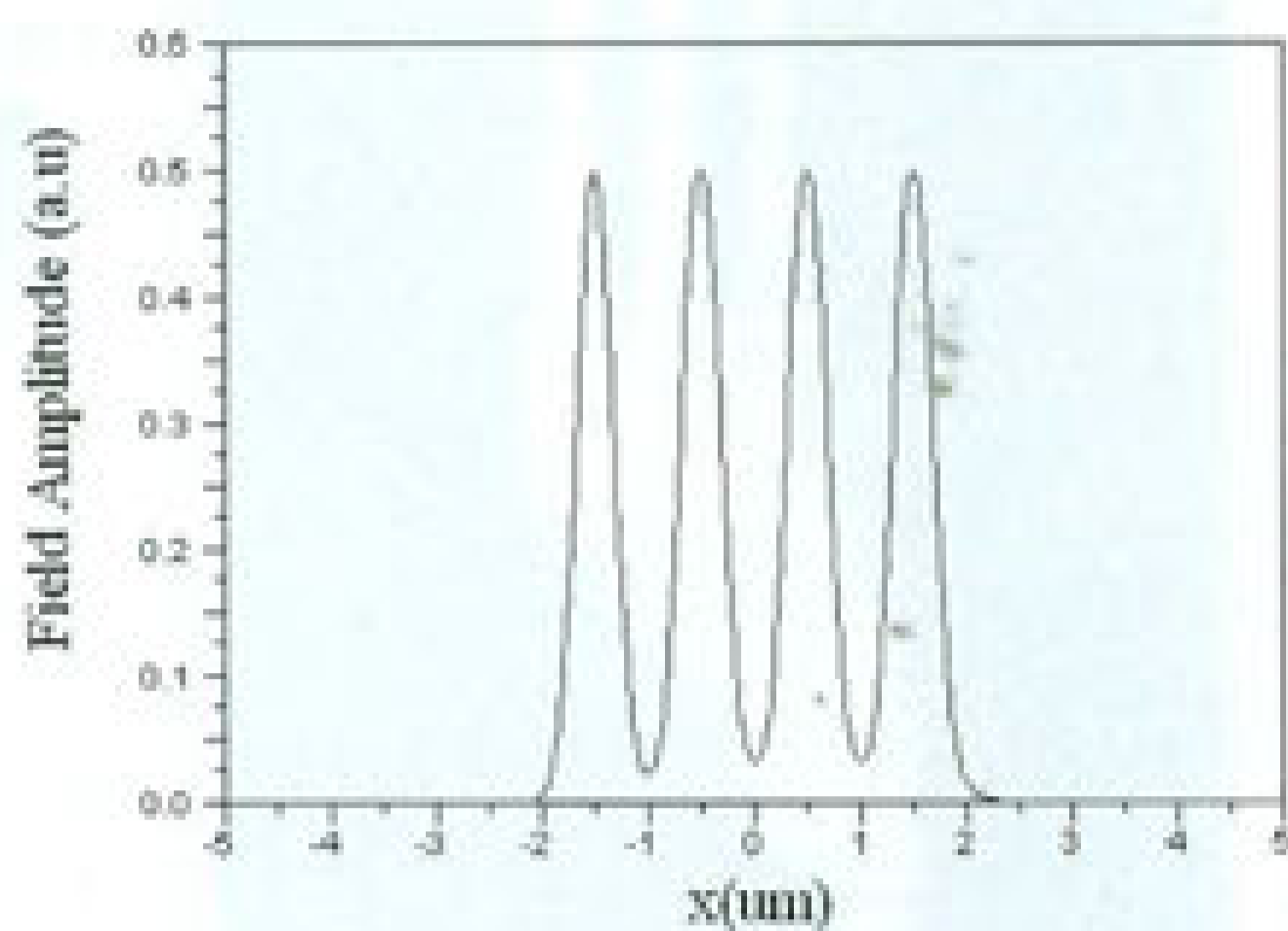
When the input vector is to be  $(x_0 x_1 x_2 x_3)^T = (1111)^T$  and the all-optical DCT-IV is performed. The normalized input and output amplitudes are shown in Fig. 6(a) and 6(b), respectively. The 3D BPM simulation for this case is shown in Fig. 6(c).

Therefore, the 3D BPM simulation results show that our theory has predicted accurately the transfer matrix of the DCT-IV. The DST-IV matrix can be verified using the similar approach.

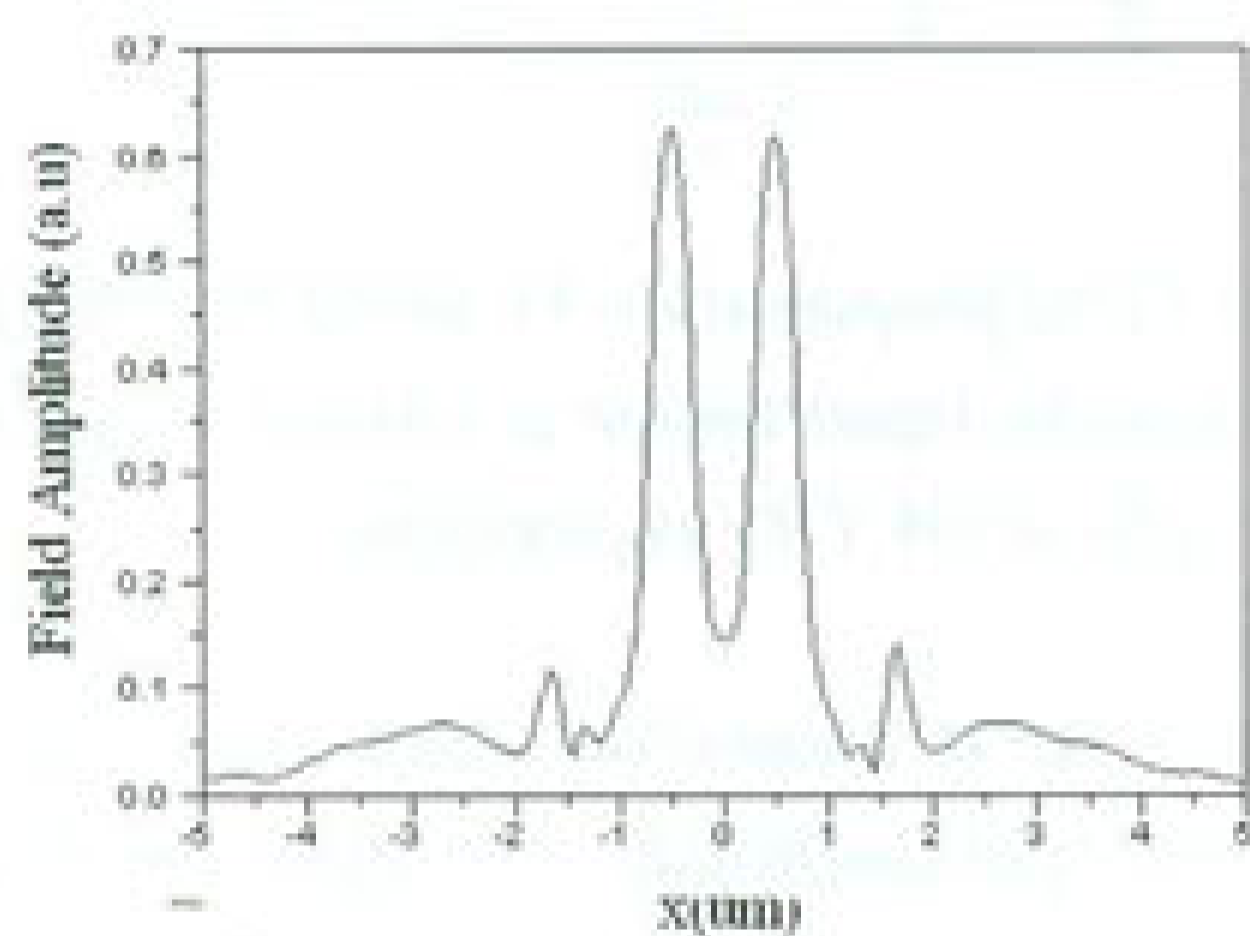
Now we will show that the sum and difference unit can be realized using  $8 \times 8$  MMI structures if the width and length of the MMI coupler are chosen properly. We choose an  $8 \times 8$  MMI coupler having a width of

$W_{MMI} = 9 \mu m$ . The 3D-BPM simulations for optimized designs of  $8 \times 8$  MMI structures based on the SOI channel waveguide having a width of  $W_{MMI} = 9 \mu m$  are shown in Fig. 7. The optimized length calculated to be  $L_{MMI} \approx 382 \mu m$ . Fig. 7 shows the input and output field of the sum and difference unit when there are two input signals presented at input port 1 and 8. The simulations show that the sum of the two signals can be obtained at output port 8 and the difference of the two signals can be obtained at output port 1. Therefore, the BPM simulation has a very good agreement with our prediction of the proposed theory.

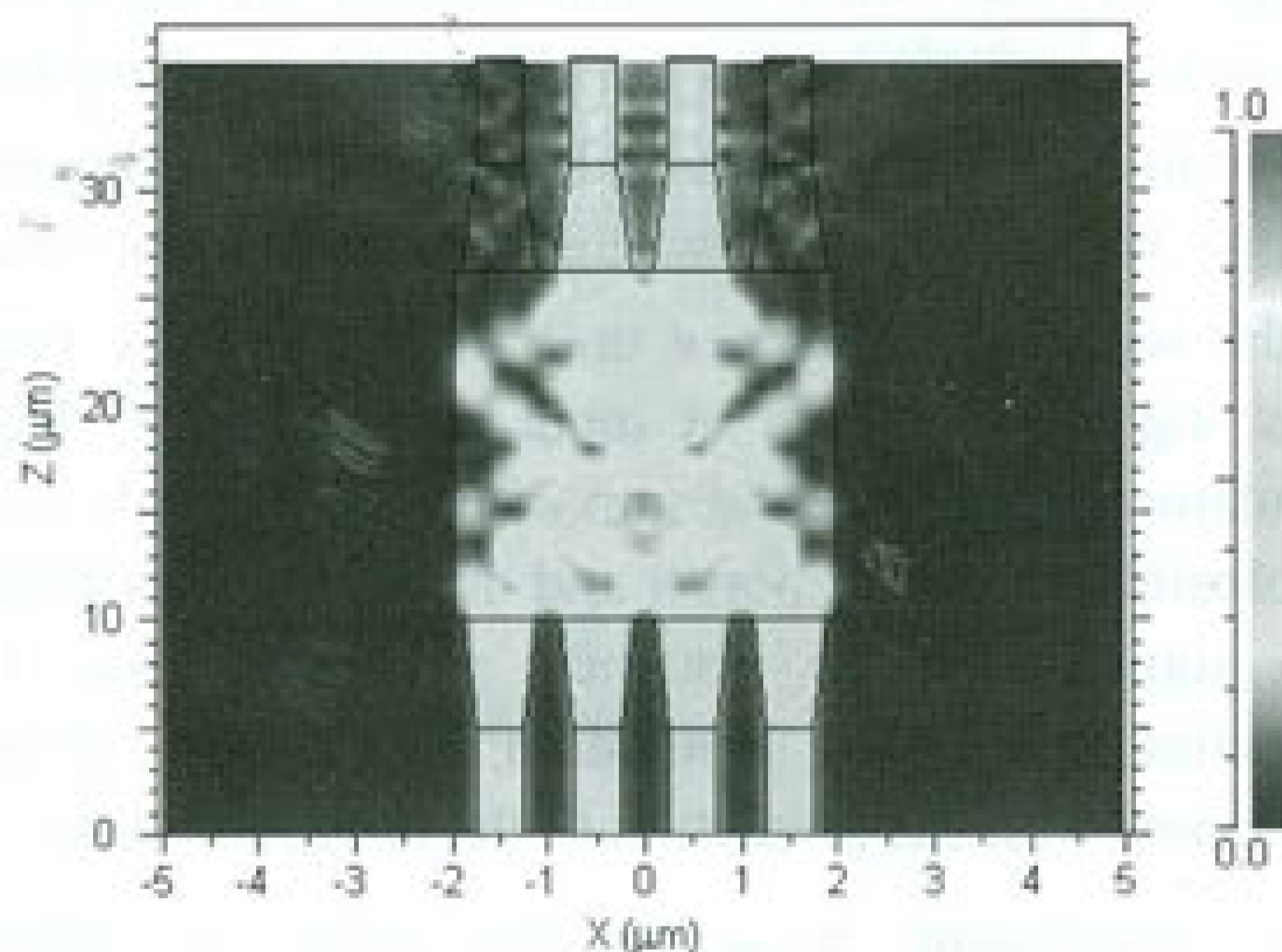
In conclusion, it is obvious from the BPM simulation results that the theory predicts adequately the field propagation in the devices. By cascading the MMI structures which have been used for the DCT-IV, DST-IV, sum and difference unit along with the phase shifters, the whole MDCT device and MDST device can be obtained.



(a) Field amplitude at the input ports for input vector  $(1111)^T$

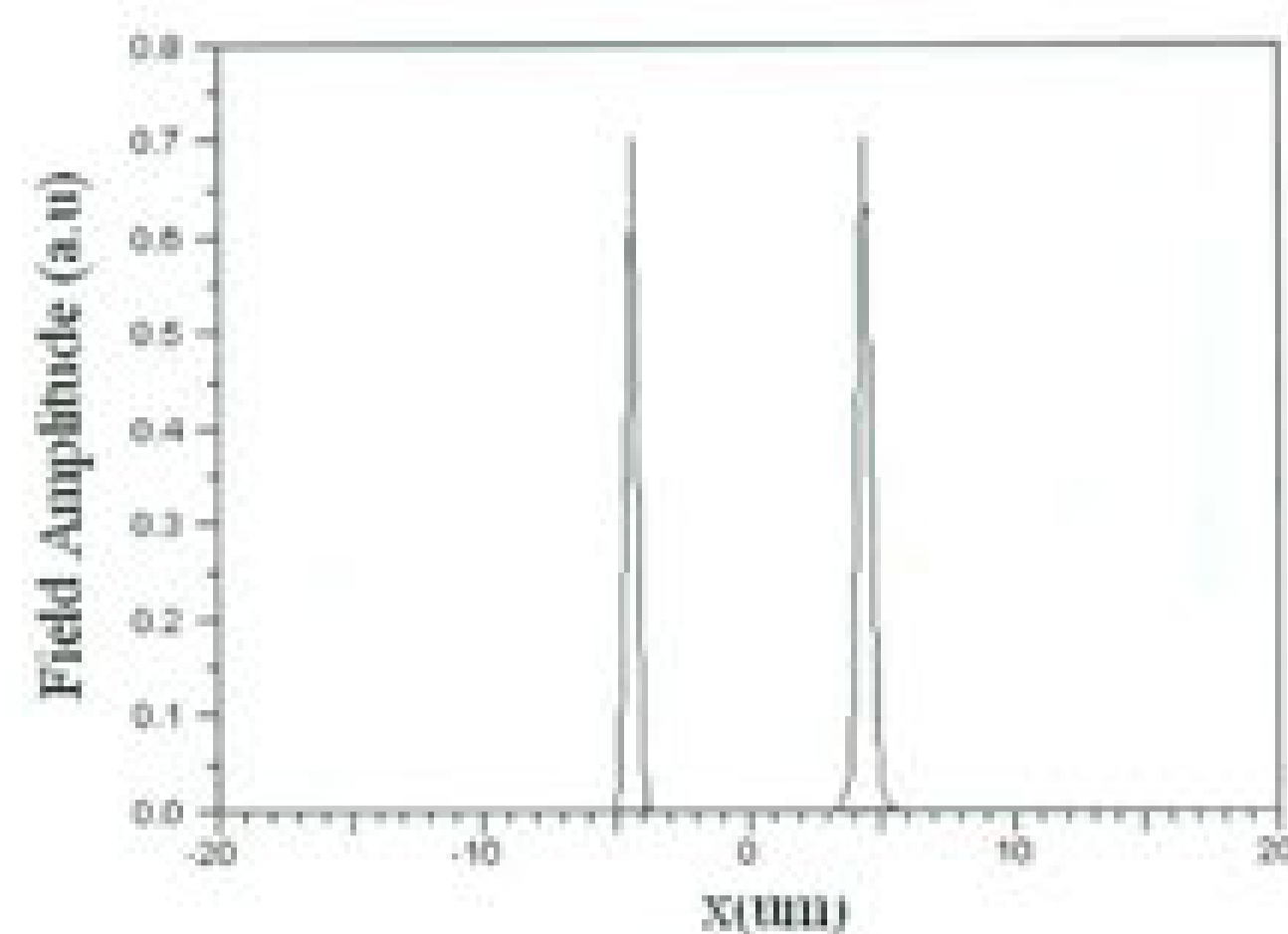


(b) Field amplitude at the output ports for input vector  $(1111)^T$  of the 4 point DCT-IV using 4x4 MMI structures

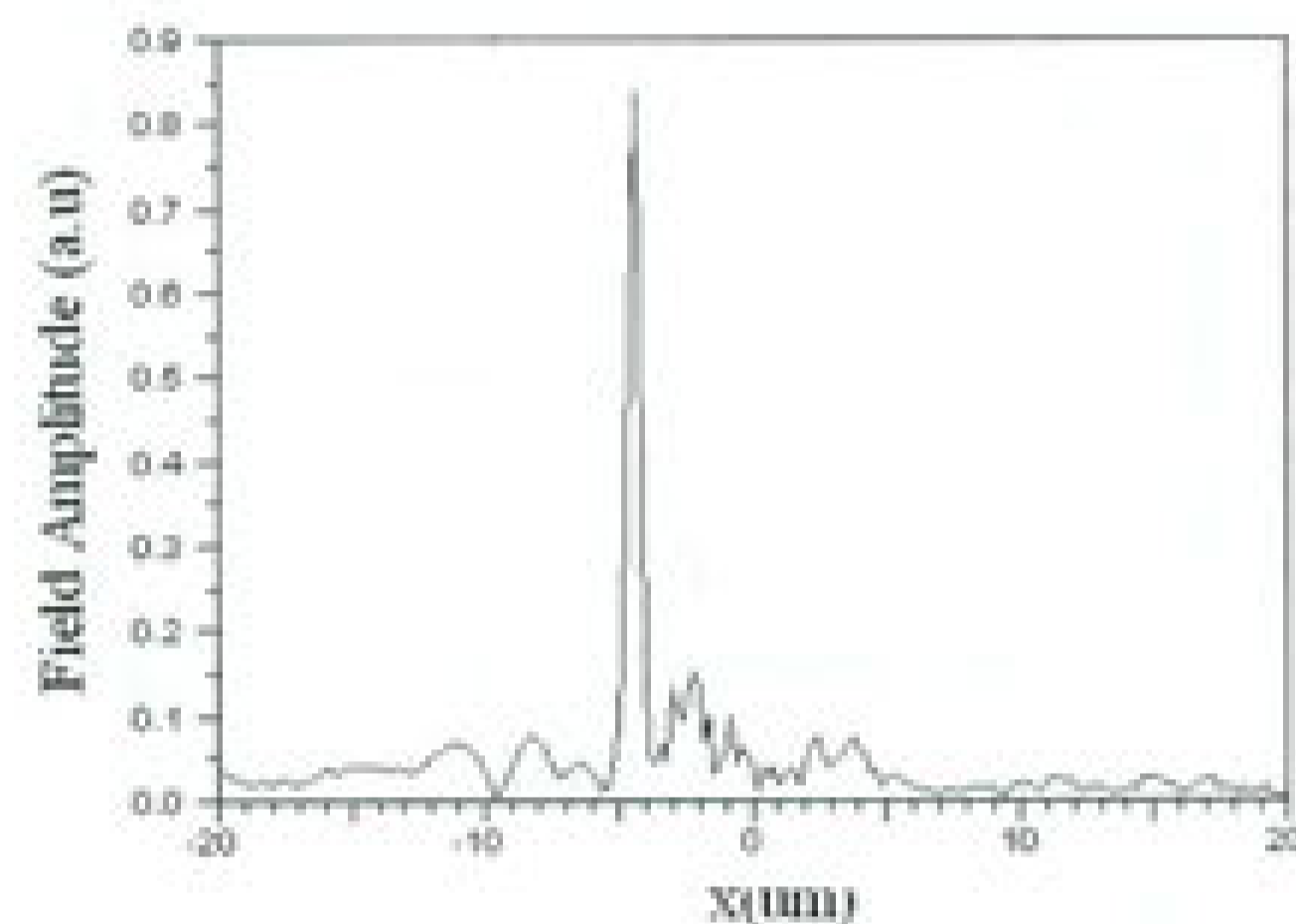


(c) Field propagation by using the BPM simulation when the input vector is  $(1111)^T$  and the all-optical DCT-IV is performed

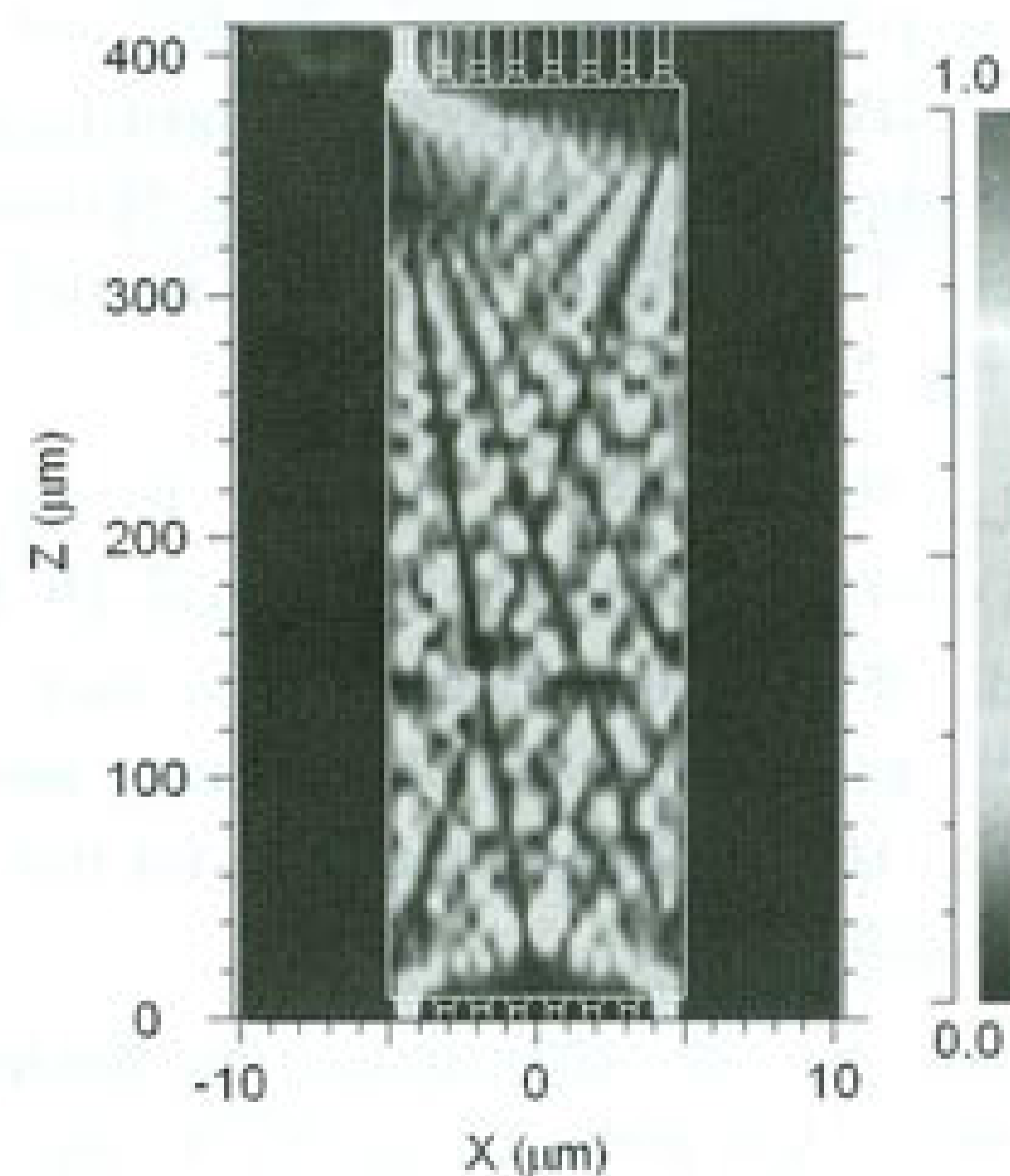
Figure 6. BPM simulation result of the DCT-IV transform: (a) input amplitude, (b) output amplitude and (c) field propagation



(a) input signals presented at two ports of the sum and difference unit



(b) output signals presented at output ports of the sum and difference unit



(c) Field propagation of the signal via the sum and difference unit using 8x8 MMI structures

Figure 7. BPM simulation result of the sum and difference unit: (a) input amplitude, (b) output amplitude and (c) field propagation

#### IV. CONCLUSION

We have proposed a method for realizing all-optical modified discrete cosine and sine transforms using multimode interference structures on an SOI platform. The designs of the proposed devices have been performed using the transfer matrix method and the beam propagation method. This all-optical approach for the MDCT and MDST realization can be useful for all-optical signal and image processing applications such as data compression, filtering and coding.

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