An Analysis of Valid Nodes Distribution for Sphere Decoding in the MIMO Wireless Communication System

Minh Thuong Nguyen[1], Xuan Nam Tran[2], Vu Duc Ngo[3], Quang Kien Trinh[2], Duc Thang Nguyen[2], Tien Anh Vu[2]
[1] Military Institute of Information Technology, Hanoi, Vietnam
[2] Le Quy Don Technical University, Hanoi, Vietnam
[3] Hanoi University of Science and Technology, Hanoi, Vietnam.
Correspondence: Minh Thuong Nguyen, thuongvcntt@gmail.com
Communication: received 25 July, revised 24 August, accepted 5 September
Digital Object Identifier: 10.32913/mic-ict-research.v2021.n2.985

Abstract: Sphere Decoding (SD) algorithms can achieve a quasi-maximum likelihood (ML) decoder performance over Gaussian Multiple Input-Multiple Output (MIMO) channels with much lower complexity compared to the exhaustive search method. The SD algorithm is based on a closest lattice point search over a limited search space (hypersphere). On top of that, QR-decomposition simplifies the SD linear system matrix to be an upper triangle matrix. The solution solver then is done by searching in the exponentially expanding search tree, started from the top with only a single node then increases by M times at every level (in $N_R \times N_T$ MIMO system). Fortunately, the SD algorithm shrinks its hypersphere at every level (once the level node is determined) and phases out a vast number of candidates, remaining only specific valid nodes in the current considered level. In this work, we propose a statistical approach for evaluating the adequate number of valid search nodes at every level of the search tree, aiming to optimize the overall computational workload. We use a massive number of inputs patterns and extensive simulation to project the number of remaining valid nodes during the searching process. The simulations have been conducted for 4×4, 6×6 and 8×8 MIMO systems. Our results indicate that for a particular targeted BER, choosing an appropriate sphere radius is essentially important and the number of necessary calculations is only large at the middle layers and can be generically quantified regardless of the system characteristics. This finding is beneficial for the hardware implementation of the SD, where the number of computational units has to be fixed in advance.

Index Terms: *Multi-input multi-output (MIMO), maximum likelihood (ML), sphere decoding (SD)*

I. Introduction

Multiple Input-Multiple Output (MIMO) has become one of the critical technologies for modern wireless communication in recent years. MIMO takes advantage of spatial diversity to provide wireless communications channels for simultaneous transmission of signal symbols, the wireless communication MIMO system can increase the capacity and spectrum efficiency [1]. Moreover, MIMO systems can improve the reliability of communication [2]. Nowadays, the MIMO system becomes a core component of HSPA+ (3G), LTE (4G), 5G, and other modern wireless communication systems. Generally, maximum-likelihood (ML) detection achieves optimal performance in MIMO systems with much higher complexity than other detection schemes. However, with the increase of transmission antennas and modulation order, the ML detection complexity increases exponentially to make it unsuitable for real-time hardware implementation.

Sphere Detection (SD) has emerged as the mainstream approach for compromising between the computational complexity and Bit-Error-Rate (BER) in MIMO decoding applied in popular systems such as Space-Division Multiplexing (SDM) and Space-Division Multiple Access (SDMA). The SD method was first proposed by Fincke and Posh [3, 4] for searching the closest vectors in the given lattice. Then, a number of SD variants have been introduced for pushing its performance close to that of ML with significantly reduced computational complexity [5–7]. The basic idea of SD is to limit the candidate set of the optimal ML solution into a hypersphere around the received signal vector. Therefore, only lattice points located inside the hypersphere are examined instead of checking all the possibilities of the transmitted signal. The search tree of SD detection can be represented as a search tree based on depth-first SD, search tree based on breadth-first K-best decoder, and breadth-first search tree based on Fixed SD [7]. Depth-first searching is a sphere decoding method with near-optimal performance close to ML decoders with moderate computational complexity for an extensive SNR range [7]. Another strategy which reduces complexity for depth-first searching is the Schnorr-Euchner (SE) decryption algorithm [8, 9]. This algorithm enumerates and sorts Euclidean distances in an ascending order for priority searching. This process converges as soon as a satisfactory solution is found in the hypersphere, thus avoiding further growth of the computational volume [11, 13]. However, the computational complexity of this decoding method depends on the chosen value of the hypersphere radius and the noise level and channel conditions. These factors have a random nature and lead to unpredictable throughput, non-deterministic latency and are unsuitable for hardware implementation. Therefore, in this work, we mainly employ the statistical analysis approach to analyze the distribution of the valid node (i.e., survival search paths in the tree) in the sphere decoder. The statistical results allow to reduce unnecessary search paths and the computational units in each level without scarifying the overall BER performance. This is especially meaningful for hardware implementation of the SE, where the structure and computing resources must be well defined in advance.

The rest of the paper is organized as follows: Section 2 presents the general system model and its corresponding signal format. In Section 3, statistical analysis of the root distribution in the sphere decoder is discussed. Section 4 discusses the SD implementation from the hardware perspective in comparison with conventional MIMO decoding schemes. Conclusions are drawn in Section 5.

II. System Model

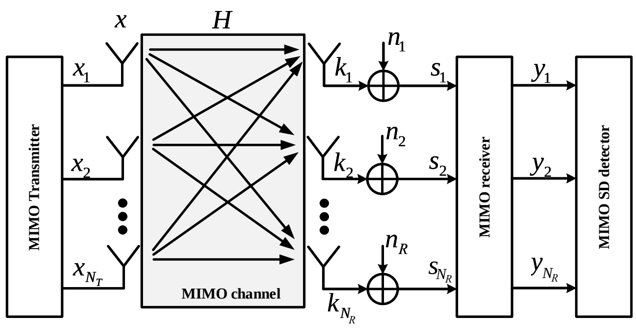


Figure 1. Block diagram of a general MIMO system.

Consider a MIMO system with N_T transmit and N_R receive antennas, as shown in Fig. 1. The MIMO channel is characterized by its complex channel matrix $\boldsymbol{H} = (h_{ij})^{N_R\times N_T} \in \boldsymbol{C}^{N_R\times N_T}$, whose elements are drawn independently from the complex Gaussian distribution with zero mean and unit variance. Those parameters indicate the attenuation and phase shift for each path from the transmitter to the receive antennas; they are assumed to be perfectly known in advance (i.e., through the channel estimation stage). For transmission, elements x_i of complex signal vector $\boldsymbol{x} = (x_i)^{N_T\times 1} \in \boldsymbol{\Omega} \subset \boldsymbol{C}^{N_T\times 1}$ are sent concurrently through N_T transmit antennas, where $\boldsymbol{\Omega}$ is the set of the signal modulation constellation. Hence, the received N_R-element complex signal vector $\boldsymbol{y} = (y_i)^{N_R\times 1} \subset \boldsymbol{C}^{N_R\times 1}$can be expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $(n_i)^{N_R\times 1} \sim CN(0, \delta^2\boldsymbol{I})$ is a complex Additive White Gaussian Noise (AWGN) vector.

The ML detector performs an exhaustive search for all the possible symbol vectors in $\boldsymbol{\Omega}$ set for obtaining the one with a minimum squared error:

$$\hat{\boldsymbol{x}}_{ML} = \arg\min_{\boldsymbol{x}\in\boldsymbol{\Omega}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\|^2. \tag{2}$$

The ML detection complexity grows exponentially with the antennas dimension. Therefore, the ML detector can be replaced with the sphere detector to reduce the computational complexity. The sphere detection only calculates the Frobenius norm for candidate points inside a hypersphere is formed around the received signal vector with a predetermined radius r_{sph}. The equation (2) can be transformed as

$$\hat{\boldsymbol{x}}_{ML} = \arg\min_{\boldsymbol{x}\in\boldsymbol{S}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\|^2, \tag{3}$$

where $\{\boldsymbol{S} \subset \boldsymbol{C}^{N_T\times 1} : \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}\| \le r_{sph}\}$ is a set of all possible points in the lattice $\boldsymbol{H}\boldsymbol{x}$, whose distance to $\boldsymbol{y}$ is always smaller than the hypersphere's radius r_{sph}. Choosing the suitable value of r_{sph} is essentially important for determining the SD's computational complexity and BER performance. To further reduce the amount of computation in SD, equation (3) can be transformed into the identical problem applying the QR decomposition (QRD) to the channel matrix, that is $\boldsymbol{H} = \boldsymbol{Q}\boldsymbol{R}$ where matrix $\boldsymbol{Q}$ is a unitary matrix whose size is $N_R \times N_R$ and $\boldsymbol{Q}\boldsymbol{Q}^H = \boldsymbol{I}$ while $\boldsymbol{R}$ is an $N_R \times N_T$ upper triangular matrix. Replacing $\boldsymbol{H}$ by $\boldsymbol{Q}\boldsymbol{R}$ and after simple transformation, equation (1) becomes:

$$\tilde{\boldsymbol{y}} = \boldsymbol{R}\boldsymbol{x} + \boldsymbol{Q}^H\boldsymbol{n}, \tag{4}$$

where $\tilde{\boldsymbol{y}} = \boldsymbol{Q}^H\boldsymbol{y}$. Note that $\boldsymbol{Q}^H\boldsymbol{n}$ has the same statistics as $\boldsymbol{n}$, hence equation (3) is equivalently characterized as

$$\hat{\mathbf{x}} = \arg\min_{\boldsymbol{x}\in\mathbf{S}} ||\tilde{\boldsymbol{y}} - \boldsymbol{R}\boldsymbol{x}||^2, \tag{5}$$

where $\hat{\boldsymbol{y}} = \boldsymbol{R}\boldsymbol{x}$. Equation (5) can be calculated through cost function as follows:

$$\mathcal{D}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) = ||\tilde{\boldsymbol{y}} - \boldsymbol{R}\boldsymbol{x}||^2 \le {r_{sph}}^2. \tag{6}$$

Since the matrix $\boldsymbol{R}$ is upper triangular, the cost function $\mathcal{D}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}})$ is also a partial Euclidean distance that can be calculated recursively from one transmit antenna to another:

$$\mathcal{D}_m(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) \triangleq \sum_{i=m}^{N_R} \left(\tilde{y}_i - \sum_{j}^{N_T} R_{ij} x_{ij} \right)^2, \tag{7}$$

$$\mathcal{D}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) = \mathcal{D}_1(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}), \tag{8}$$

$$\mathcal{D}_{m-1}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) = \mathcal{D}_m(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) + \left(\tilde{y}_{m-1} - \sum_{i=m-1}^{N_T} R_{m-1,i} x_i \right)^2, \tag{9}$$

where $\tilde{y}_{m-1}$ is the $(m-1)^{th}$ element of the received signal vector after multiplication of the received signal vector by $\mathbf{Q}^H$; $R_{i,j}$ is an entry of matrix $\boldsymbol{R}$ that belongs to the i^{th} row and the j^{th} column, and the cost function $\mathcal{D}_m(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}})$ is a partial Euclidean distance of the candidate symbol $\boldsymbol{x}$ at the search level m-th. For all possible transmit symbol vectors that are satisfied $\boldsymbol{x} \in \{\boldsymbol{S} \subset \boldsymbol{C}^{N_T \times 1} : \|\boldsymbol{R}\boldsymbol{x} - \boldsymbol{y}\| \le r_{sph}\}$, we set $\mathcal{D}_{N_R+1}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) = 0$, and have the following inequality:

$$\mathcal{D}_{m-1}(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) \le {r_m}^2 - \mathcal{D}_m(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}}) \tag{10}$$

where ${r_m}^2 = {r_{sph}}^2 - \sum_{i=m+1}^{N_R} \mathcal{D}_i(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{y}})$ and $m = N_T, N_T - 1 ... 2$.

For hardware implementation, it is also more efficient to perform a Real-Valued Decomposition (RVD) of $\boldsymbol{H}$, which simplifies the computation of the Euclidean distance. The RVD decouples the system equation (1) into a new real-valued representation as follows [12, 15]

$$\begin{bmatrix} \Re(\boldsymbol{y}) \\ \Im(\boldsymbol{y}) \end{bmatrix} = \begin{bmatrix} \Re(\boldsymbol{H}) & -\Im(\boldsymbol{H}) \\ \Im(\boldsymbol{H}) & \Re(\boldsymbol{H}) \end{bmatrix} \begin{bmatrix} \Re(\boldsymbol{x}) \\ \Im(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} \Re(\boldsymbol{n}) \\ \Im(\boldsymbol{n}) \end{bmatrix} \tag{11}$$

In (11), $\Re(.), \Im(.)$ denote the real and imaginary parts of a complex number, respectively. So we can solve equation (1) through equation (11) with the above steps with transforming the complex constellation set into an integer set as follows:

$$\boldsymbol{S}_r = \left\{ -\sqrt{M} + 1, ..., \sqrt{M} - 1 \right\} \tag{12}$$

where M is the modulation order. The QRD can then be performed generally based on the augmented channel equation in (12). The size of the equivalent channel matrix ($\boldsymbol{H}$), the unitary matrix ($\boldsymbol{Q}$) and the upper triangular matrix ($\boldsymbol{R}$) are transformed to $2N_R \times 2N_T$, $2N_R \times 2N_R$, $2N_R \times 2N_T$, respectively. The number of levels of tree search change from $2N_R$ to 1. In the following section, we are going to analyze the solution distribution in the sphere decoder.

III. Statistical Analysis of the Valid Node Distribution in SD Search Tree

In the SD algorithm, the number of satisfied solutions in the sphere at each layer is randomly distributed and depends heavily on the model parameters, including the sphere radius r_{sph} and the Signal-to-Noise (SNR) ratio. Indeed, choosing too large r_{sph} increases the number of points inside the hypersphere while setting a small r_{sph} essentially constraints the number of candidate points. The latter increases the probability that the correct solution is laying outside the sphere. On the other hand, the former reduces the chances of taking the wrong output vector at the cost of higher computation and decoding latency. Furthermore, the SNR ratio inherently affects the SD result, i.e., the larger (smaller) SNR improves (degrades) the algorithm's accuracy (degrades).

This section analyzes the distributions of the number of valid nodes (i.e., survival search paths) at every SD algorithm's step with respect to different radius sizes and SNR. A conventional SD model has been built on MATLAB for 4×4, 6×6 and 8×8 antennas MIMO systems. The SD model would search for every possible path in its tree for statistically logging and building the node's distribution in each level. We generate a massive number of random inputs (1 million test vectors) of the 16-QAM modulation signal that ensure reliable evaluation in the statistical sense. Each element of the equivalent channel matrix $\boldsymbol{H}$ is normalized to follow the standard Gaussian distribution. AWGN was adopted for emulating the noise components in the models, and the E_b/N_0 varies from 0 to 15dB while the radius varies from 1 to 8, considering the matrix element and input signals are normalized in the range $[-1, 1]$.

The simulation results for 4×4, 6×6, and 8×8 MIMO systems are reported in Fig. 2, 3 and 4, respectively. The x-axis denotes the level in the search tree, where 8, 12 and 16 correspond to the root (the top) level in the tree[1]. The y-axis represents the mean (μ_{node}) and the standard deviation (σ_{node}) of the valid nodes remain in every layer statistically evaluated for a sufficiently large number of random inputs for each (r and E_b/N_0) pair. Here, 4×4 MIMO has 8 search tree levels, 6×6 MIMO has 12 search tree levels, and 8×8

[1]The evaluation is considered reliable when increasing the number of random patterns having little impact its results.

MIMO has 16 search tree levels, corresponding to the rank of the matrix $\boldsymbol{H}$.

From Figs. 2, 3 and 4, some notable trends could be observed for three models. First, increasing the sphere radius r_{sph} leads to an increase in both the μ_{node} and σ_{node} (i.e., the spreading level of the valid node) in every level for 4×4, 6×6, and 8×8 MIMO systems. As mentioned earlier, when r_{sph} is too small, there may be no solutions inside the sphere, making it impossible to locate the correct solution, especially in the scenarios with high E_b/N_0. On the other hand, when r_{sph} is too large, the possible solution may surge up, leading to a highly inefficient hardware design to search for the correct transmitted symbol. Therefore, it is a critical design knob to set an adequate level of r_{sph} to balance the hardware efficiency and the BER performance. This tendency is also observed for the impact of E_b/N_0, i.e., increase E_b/N_0 implies a similar but weaker impact on the μ_{node} and σ_{node}.

Furthermore, the maximum of the curve of the mean and the standard deviation of the valid nodes according to the level number in the search tree depends mainly on the sphere's radius. By selecting a sub-optimal radius, the maximum value of the valid solutions always lies in the middle layers of the search tree and gets smaller in the lower layers. Inversely, when we choose a huge sphere radius, the number of valid nodes at the lower layers increase exponentially with the level number. This can be explained by the fact that the number of nodes exponentially expanded while lowering the search level and the while the shrinking the hypersphere at the same time limits the search set. In detail, the solution vector is not entirely determined at the top level, so the possible choices are small. The search tree tends to expand rapidly at a higher layer, but as soon as a component in the solution vector is determined, the sphere dimension and its corresponding effective radius are reduced (see eq. (10)). As a result of the combined effects, the peak is typically found at the middle level.

It is also noticeable that SD in a 8×8 MIMO system is much computationally intensive compared to the 4×4 MIMO system, where the number of survival search paths is 10 times higher in both mean and standard deviation. The high σ_{node} indicates that in the worst-case scenarios, the number of search paths could be very large. Indeed, let us consider 4×4 MIMO systems at the moderate sphere radius ($r_{sph} = 3$) with E_b/N_0 ranging from 0 dB to 15 dB, the worst-case number would reach 328. From the practical point of view, it is a considerable saving in computational volume by not taking the worst case but limiting a specific search path coverage probability. From the statistical results, we calculate the number of valid nodes for 99.99% coverage[2] at the most computing-intensive level ($3-6$ for 4×4, $7-10$ for 6×6 and $8-12$ for 8×8 antennas) and report it in Fig. 5. From the figure, the number of required search nodes in both cases are also found weakly dependent on the E_b/N_0 as expected. Also, the number of search path required for a 4×4 antenna is much lower than that for 8×8 configuration. Nonetheless, by not taking the worst case the number of paths is about $\sim 10-14$ times lower than the worst-case (i.e., 20-30 paths compared to 328), this could be explained by the fact that σ_{node}/μ_{node} is observed large ($\sim 80-125\%$) in most of the cases (see Figs. 2 and 3). The impact of sphere radius r_{sp} is also notable here, i.e., number of require search paths increases mostly exponentially with r_{sp}.

The chosen spherical radius significantly influences the number of valid nodes at each level, as shown in the Figs. 2, 3 and 4. The ML detection could be viewed as the particular case of the SD algorithm with an infinite radius value. In such case, the number of nodes to be traversed at the k-th level is $M^{(2N_R+1-k)}$, where M is the modulation order, N_R is the number of receiving antennas in the system, and k receives value from $2N_R$ to 1. So, with a large value of r_{sph}, the number of valid nodes from i^{th} level to $(i-1)^{th}$ level increases by M times (i.e., exponentially). Thus, the larger the MIMO system, the more complex the system becomes. The decoder will have to perform an extremely large number of calculations for such MIMO systems, which is not feasible for both software and hardware implementations in practice. Our statistical analysis points out an important fact that by selecting an appropriate constraint r_{sph}, the SD could improve the searching latency by a few orders of magnitude while delivering similar level of BER compared to the ML detection. For example, for 4×4, 6×6, and 8×8 MIMO systems, SD with choosing $r_{sph} = 3$, the number of search nodes at the critical layer 5, 8, and 11 reduces by ~ 11, ~ 52, and ~ 293 times, respectively, compared to the total number of nodes in those levels.

IV. Further Discussion on Hardware Implementation of SD

Hardware implementation is typically much faster than the software and required for real systems, but it must accept the minimum flexibility. Therefore, the SD design must be rigorously optimized to balance multiple objectives such as decoding latency, energy, and the bit error rate and the hardware resource in advance. Although some other works proposed methods for optimizing the search tree to reduce

[2]The number search node that with probability of 99.99 % will cover all the possible search node for all 1 million testcases. This value is extracted from the cumulative distribution function (CDF) plots as illustrated in Fig.4.

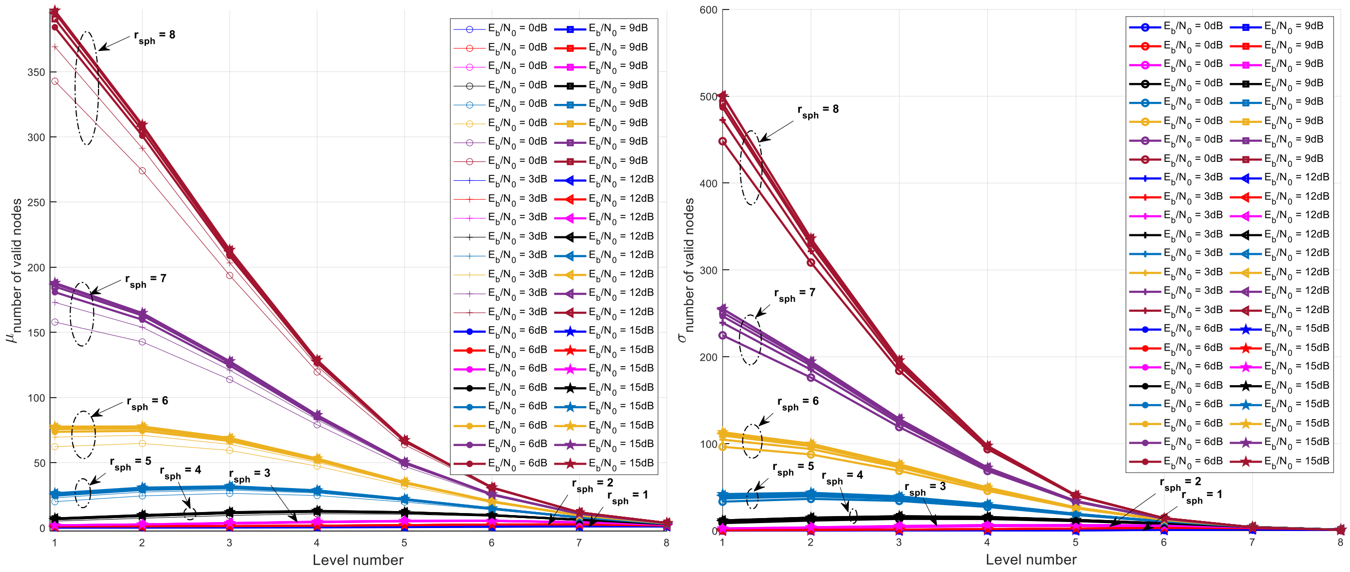


Figure 2. Mean (μ_{node}) and standard deviation (σ_{node}) of valid search nodes in each level of SD search tree in 16-QAM 4 × 4 MIMO system, simulated for 1 million random input pattern with AGWN.

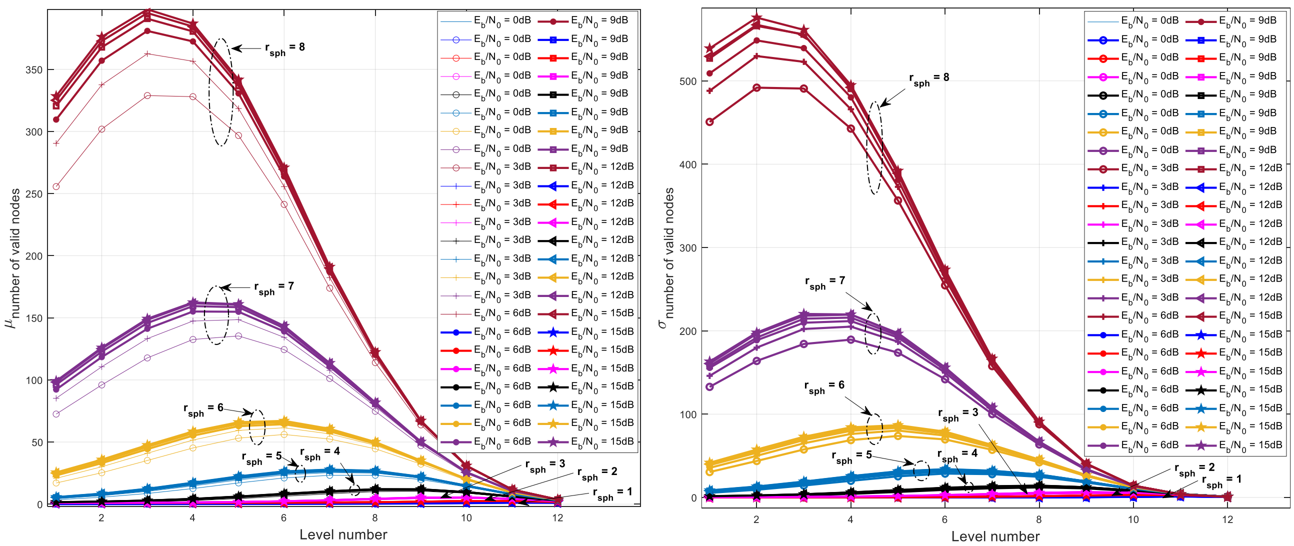


Figure 3. Mean (μ_{node}) and standard deviation (σ_{node}) of valid search nodes in each level of SD search tree in 16-QAM modulation 6 × 6 MIMO system, simulated for 1 million random input pattern with AGWN.

the computation effort with the acceptable trade-off in BER [14–16]. However, those optimization approaches either is empirically or have been done for software implementations rather than for hardware ones. In this work, we provide the entire projection of the statistical distribution of the valid search node for the SD algorithm. This projection solely depends on the antenna configuration and could give some essential knobs for hardware implementation of the SD algorithm.

From the statistical results in Section 3, the spherical radius is an important parameter that essentially determines the system complexity or the computational workload. The smaller r_{sph} the faster the SD implementation. However, using small r_{sph} intuitively would degrade the BER as it increases the chance that the actual solution populates outside the sphere. We use the same system model in Section 3 to quantize the BER performance of the SD for r_{sph} and E_b/N_0 ranging from 1 to 8 and $0 - 15$ dB, respectively. We also compared the SD results with other popular algorithms, including Zero Forcing (ZF) [17, 18] and Minimum Mean Square MMSE [17, 18] with the same

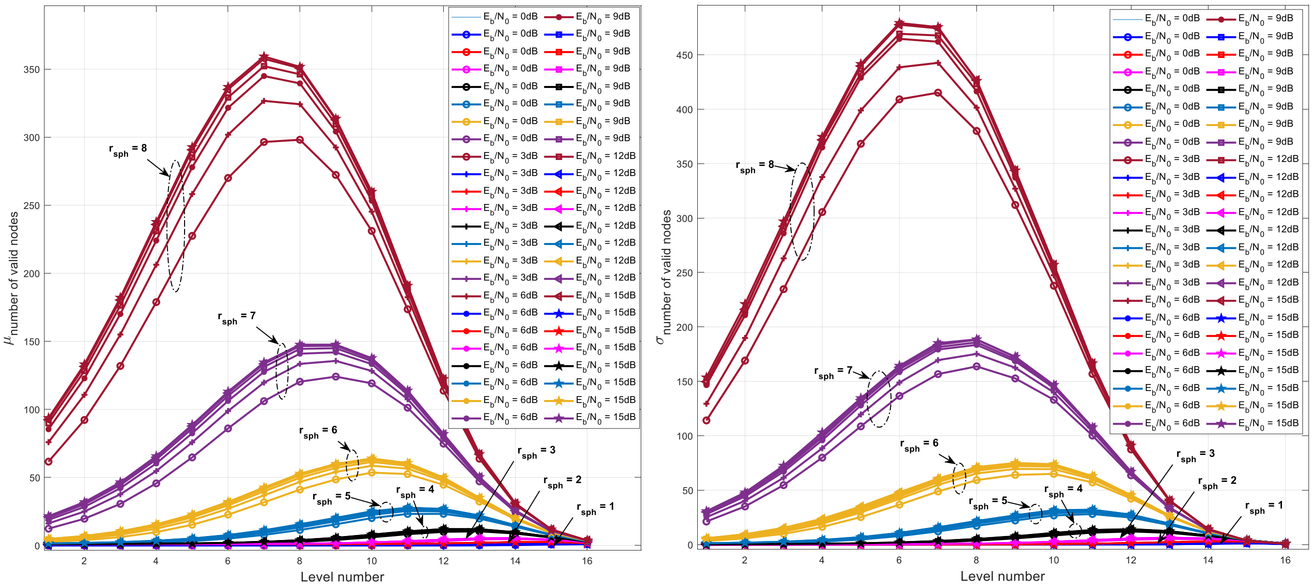


Figure 4. Mean (μ_{node}) and standard deviation (σ_{node}) of valid search nodes in each level of SD search tree in 16-QAM modulation 8×8 MIMO system, simulated for 1 million random input pattern with AGWN.

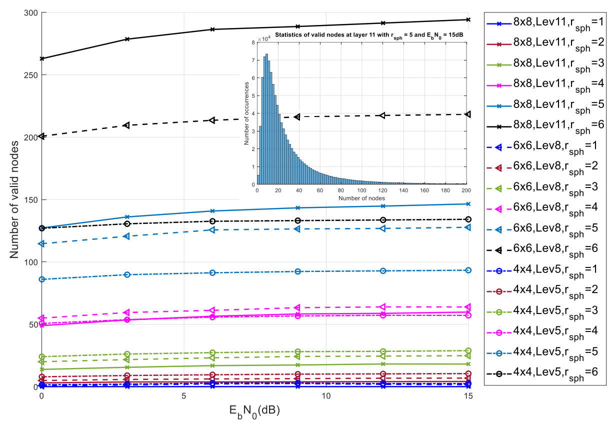


Figure 5. The number of valid nodes in the most intensive computing level for 4×4, 6×6 and 8×8 MIMO systems, extracted from the CDF corresponding to 99.99 % coverage of the total available search paths

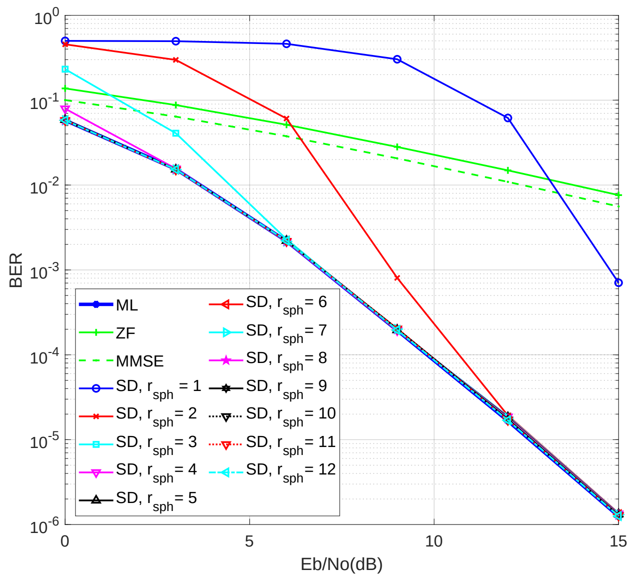


Figure 6. BER performance comparison among detection algorithms for 4×4 MIMO, simulated for 1 million random input patterns with AGWN

inputs pattern and channel model as presented in Fig. 5.

From Fig. 5, it is clear that ZF and MMSE, which are very lightweight algorithms (i.e., suitable for hardware implementation), have much lower performance than the baseline ML, e.g., BER of ZF and MMSE is quite similar and is higher than BER of ML by a few orders of magnitude. Regarding the SD performance, the simulation results show that SD shows very poor performance with a small radius ($r_{sph} \leqslant 2$). However, the BER of SD with $r_{sph} = 3$ and above approach the baseline ML BER. Very similar results were observed for 8×8 MIMO (the results are skipped here for the sake of brevity). Therefore, our results confirm an important observation that the optimum value of r_{sph} could reduce significantly computational workload in the SD algorithm while maintaining a fairly good level of BER.

Also, from the analysis in Section 3, the computational volume of the system is found to be heavy somewhere in the middle levels. Limiting the solutions at first and the last layers will certainly not bring any benefit. Therefore, it is a good practice to support full search nodes in the top and last levels, and the number of valid nodes in those levels is typically low. Hence, the hard decision remains for the middle layer, where it is a must to find the optimum (i.e., the lowest) number of computation units to match the system performance. This optimization essentially must be done in conjunction with the sphere radius optimization discussed above. For instance, with $r_{sph} \leqslant 3$ the necessary number of computation units is about μ_{node} (5 for 4×4 system and 10 for 8×8 system), considering other techniques for local pruning/filtering, the search paths are applied. As a result, the computation workload of the SD implementation could be much lower than that of the ML, which is important to make the design practical.

V. Conclusion and future works

The sphere decoder and similar tree search algorithms provide an excellent performance-complexity trade-off, making them suitable for hardware implementation of the MIMO communication system. The tree search algorithms have been extended to iterative decoding, allowing improved error rate performances. Nonetheless, the advanced 5G standard and the increase of the MIMO antenna dimension pose new technical challenges to the implementation of 5G devices in general and the channel decoder. Using the statistical analysis approach, we could provide both qualitative and quantitative results, which give insights into how the SD algorithm can be optimized. It is a notable finding that choosing an appropriate sphere radius and adequately limiting the computation node at the critical search tree level could effectively balance the BER performance and computational/hardware efficiency. Based on those important findings, we will work toward to the sub-optimized solution for SD realization that may include approaches for narrowing/pruning search paths, dynamic resource allocation, and micro-architecture optimization targeted for hardware accelerator applied in the practical MIMO systems.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585-596, 1999.

[2] X. Chen, S. Zhang and Q. Li, "A Review of Mutual Coupling in MIMO Systems," *IEEE Access*, vol. 6, pp. 24706-24719, 2018.

[3] U. Fincke, M. Pohst, "Improved methods for calculating vectors of short length," *Mathematics of Computation*, 1985.

[4] M. Pohst, "On the computation of lattice vectors of minimal length, successive minima," *SIGSAM Bull.*, vol. 15, no. 1, pp. 37-44, 1981.

[5] H. Jang, S. Nooshabadi, K. Kim and H. Lee, "Circular Sphere Decoding: A Low Complexity Detection for MIMO Systems With General Two-dimensional Signal Constellations," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 3, pp. 2085-2098, March 2017.

[6] W. Hwang, J. Park, J. Kim and H. -S. Lim, "Optimal Precoder Selection for Spatially Multiplexed Multiple-Input Multiple-Output Systems With Maximum Likelihood Detection: Exploiting the Concept of Sphere Decoding," *IEEE Access*, vol. 8, pp. 223859-223868, 2020.

[7] J. Liu, S. Xing and L. Shen, "Lattice-Reduction-Aided Breadth-First Tree Searching Algorithm for MIMO Detection," *IEEE Communications Letters*, vol. 21, no. 4, pp. 845-848, April 2017

[8] C. P. Schnorr and M. Euchner, "Lattice Basis Reduction: Improved Practical Algorithms and Solving Subset Sum Problems," *Math. Program*, vol. 66, p. 181-191, 1994.

[9] X. Nguyen, M. Le, N. Pham and V. Ngo, "A pipelined Schnorr-Euchner sphere decoder architecture for MIMO systems," *2015 International Conference on Advanced Technologies for Communications (ATC)*, Ho Chi Minh, Viet Nam, 2015.

[10] B. Shim and I. Kang, "Sphere Decoding With a Probabilistic Tree Pruning," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4867-4878, Oct. 2008

[11] K. Nikitopoulos, A. Karachalios and D. Reisis, "Exact Max-Log MAP Soft-Output Sphere Decoding via Approximate Schnorr–Euchner Enumeration," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 6, pp. 2749-2753, June 2015

[12] I.A. Bello, B. Halak, M. El-Hajjar, et al. "VLSI Implementation of a Fully-Pipelined K-Best MIMO Detector with Successive Interference Cancellation" *Circuits Syst Signal Process 38*, 4739-4761, 2019.

[13] Y. Ding, N. Li, Y. Wang, S. Feng and H. Chen, "Widely Linear Sphere Decoder in MIMO Systems by Exploiting the Conjugate Symmetry of Linearly Modulated Signals," *IEEE Transactions on Signal Processing*, vol. 64, no. 24, pp. 6428-6442, 15 Dec.15, 2016

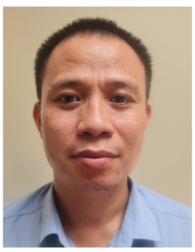
[14] X. Jun, G. Diyuan and W. Zengye, "Research of Improved Sphere Decoding Algorithm," *2019 Chinese Control And Decision Conference (CCDC)*, pp. 1043-1047, 2019.

[15] B. Shim and I. Kang, "Sphere Decoding With a Probabilistic Tree Pruning," *IEEE Transactions on Signal Processing*, vol. 56, pp. 4867-4878, Oct. 2008.

[16] Y. Sonoda and H. Zhao, "Improved sphere decoding algorithm with low complexity for MIMO systems," *2014 IEEE/CIC International Conference on Communications in China (ICCC)*, pp. 11-15, 2014.

[17] M. -T. Nguyen, V. -D. Ngo, X. -N. Tran and M. -T. Le, "Design and Implementation of Signal Processing Unit for Two-Way Relay Node in MIMO-SDM-PNC System," 2019 26th International Conference on Telecommunications (ICT), 2019

[18] Trần Xuân Nam, Lê Minh Tuấn, "Xử lý tín hiệu không gian thời gian", *Nhà xuất bản khoa học và kỹ thuật*, 2013



Minh Thuong Nguyen is currently a researcher at Military Institute of Information Technology, Viet-nam. He received his B.Ege. and M.Sc degrees both in electronic engineering from Le Quy Don Technical University, Viet-nam, in 2004 and 2012 respectively. He is working toward his Ph.D degree in electronic engineering at the Military Institute of Science and Technology. His research interest includes space-time signal processing for communications, integrated circuit design.

Email: thuongvcntt@gmail.com

Xuan Nam Tran is currently a professor and head of the research group in advanced wireless communications in Le Quy Don Technical University Vietnam. He received his Master of Engineering (ME) in Telecommunications Engineering from University of Technology Sydney, Australia in 1998, and Doctor of Engineering in Electronic Engineering from The University of Electro-Communications, Japan in 2003. Dr. Tran's research interests are the areas of space-time signal processing for communications such as adaptive antennas, space-time coding, MIMO, spatial modulation and cooperative communications.
Email: namtx@mta.edu.vn

Vu Duc Ngo is currently a lecturer at School of Electronics and Telecommunications, Hanoi University of Science and Technology, and a researcher at MobiFone Research and development Center, MobiFone corporation, Vietnam. He received the Ph.D. degree from Korea Advanced Institute of Science and Technology in 2011. His research interests are the fields of SoC, NoC design and verification, and VLSI design for multimedia codecs as well as wireless communications PHY layer. Dr. Ngo is the recipient of IEEE 2006 ICCES and IEEE 2012 ATC best paper awards. Dr. Ngo is a member of IEEE.
Email: duc.ngovu@hust.edu.vn

Quang Kien Trinh received BS (2007) and MS (2009) Degrees of Applied Mathematics and Physics respectively from Institute of Physics and Technologies (State University) in Moscow, Russia; he received PhD (2018) in Computer Engineering from National University of Singapore. Currently, he is a senior researcher and the deputy head of the Department of Microelectronics and Microprocessor Engineering, Faculty of Radio-Electronics, Le Quy Don Technical University. His research interest includes low-power integrated circuit design, emerging memory technologies, and hardware security.
Email: kien.trinh@lqdtu.edu.vn

Duc Thang Nguyen is currently a final year student at Le Quy Don Techincal University. His current researches are on space-time signal processing for communications and VLSI, low-power integrated circuit design.
Email: ducthang98mta@gmail.com

Tien Anh Vu is currently a final year student at Le Quy Don Techincal University. His current researches are on space-time signal processing for communications and VLSI, low-power integrated circuit design.
Email: vtienanhmta@gmail.com